Journal of Atmospheric and Solar-Terrestrial Physics 179 (2018) 342–357

Contents lists available at ScienceDirect

Journal of Atmospheric and Solar-Terrestrial Physics

journal homepage: www.elsevier.com/locate/jastp

Atmospheric gravity wave ray tracing: Ordinary and extraordinary waves

R. Michael Jones*, Alfred J. Bedard

Cooperative Institute for Research in Environmental Sciences, University of Colorado, Boulder, CO, 80309-0216, USA

ARTICLE INFO

Keywords:
Internal gravity waves
Dispersion relation
Hamiltonian ray tracing

ABSTRACT

Calculation of internal gravity-wave ray paths in the atmosphere using a general three-dimensional ray tracing computer program is discussed. To initialize a ray-path calculation, it is necessary to specify the initial values for the components of the wave vector at the source so that the dispersion relation is satisfied. For acoustic waves, or for gravity waves in the absence of wind, there is no ambiguity in determining the magnitude of the wave vector once the frequency and direction of propagation (wave-normal direction) have been specified. For gravity waves with wind, however, it is necessary to solve a quartic equation to specify the magnitude of the wavenumber. Two of the roots of the quartic reduce to the usual solutions in the absence of wind, and we designate these roots as 'ordinary waves.' The two new roots (whose values approach infinity as the wind speed approaches zero), we designate as 'extraordinary waves.' A section contrasts the properties of the different gravity wave types.

Comparison with some previously published examples of ray-path calculations of gravity waves shows that those previous examples were actually extraordinary waves, but their significance was not recognized at that time.

In the absence of wind, gravity waves are restricted to a fan of propagation directions centered about horizontal propagation, but asymptotic gravity waves (in the Bousinesq approximation) have a fixed propagation direction for a given frequency. In the presence of a horizontal wind, however, the wave-normal direction is restricted to a fan of directions in the upwind direction, but not in the downwind direction. The ray direction for upwind propagation has no restrictions. A short review of gravity wave theory and the gravity wave dispersion relation is included.

1. Introduction

Gravity waves are ubiquitous in the atmosphere because they are generated by earthquakes, tsunamis, volcanic eruptions, severe storms, and nuclear explosions, in addition to less spectacular mechanisms (e.g. situations where the Richardson number is less than 1/4). See, for example, the work by Stenflo (1986, 1987, 1991, 1996); Stenflo and Stepanyants (1995); Jurén and Stenflo (1973) and the review by Fritts and Alexander (2003, 2012). Section 2 reviews the broad range of atmospheric gravity wave generation mechanisms and their areas of practical impacts.

Atmospheric gravity waves can be detected directly using arrays of pressure sensors (e.g. Liu et al., 1982; Bedard, 1984; Bedard et al., 2004; Bedard, 1982) or can be detected indirectly by using Lidar measurements of temperature profiles (e.g. Chen et al., 2013, 2016; Zhao et al., 2017; Chu et al., 2018) or by measuring the effects of gravity waves in the ionosphere (Nekrasov et al., 1995; Rolland et al., 2010; Makela et al., 2011; Hickey et al., 2009; Artru et al., 2005) or troposphere (Arai et al., 2011; Hickey et al., 2009). Gravity waves can also be detected by radar, rocket soundings, and satellites. In the case of gravity wave propagation to the ionosphere, it is necessary to consider the effect of the Earth's magnetic field (Nekrasov et al., 1992, 1995; Pokhotelov et al., 1994, 1996, 1998; Chmyrev et al., 1991; Streltsov et al., 1990) and to use a dispersion relation for magneto-acoustic-gravity waves (Ostrovsky, 2008; Jones et al., 2017).

Determining the sources of atmospheric gravity waves and understanding how atmospheric gravity waves are generated by their sources is an area of active research. Using ray tracing to help interpret the propagation of gravity waves helps in that endeavor and in part has motivated our work. For example, we are using ray tracing to help interpret the gravity waves observed by Lidar measurements of temperature profiles in Antarctica (Chen et al., 2013, 2016; Zhao et al., 2017).

Section 3 discusses ray tracing, including giving the dispersion relation we used in our ray path calculations. Section 4 shows propagation effects of the Acoustic cutoff frequency and the Brunt-Väisälä frequency. Section 5 discusses non-asymptotic gravity waves. Section 6 shows how the ray tracing program calculates the magnitude of the

* Corresponding author.
E-mail address: michael.jones@colorado.edu (R.M. Jones).

https://doi.org/10.1016/j.jastp.2018.08.014
Received 14 May 2018; Received in revised form 6 August 2018; Accepted 27 August 2018
Available online 30 August 2018

wave vector to initialize the calculation of a ray path. In the presence of wind, there is an additional difficulty in initializing wavenumber, because in the presence of wind there are two types of waves, which we refer to as "ordinary" and "extraordinary." Section 7 shows dispersion-relation diagrams for gravity waves with and without wind. The dispersion-relation diagrams for the extraordinary waves are very different from those for the ordinary waves. Section 8 shows the effect of including wind on the propagation of gravity waves. Extraordinary wave ray paths are very different from those for ordinary waves. Section 9 compares our ray path calculations and analysis with gravity-wave ray-path calculations of others. We show that some previously published examples of gravity-wave ray paths are actually extraordinary waves. Section 10 discusses properties of various gravity wave types. Section 11 discusses meteorological applications of a gravity wave ray tracing program. Section 12 gives some concluding remarks.

2. Impact areas of atmospheric gravity waves

The areas listed below, with examples, reflect the broad impacts of atmospheric gravity waves in terms of being generated by a rich variety of phenomena, initiating geophysical events, and transferring energy, permitting interactions between the lithosphere, atmospheric layers and the ionosphere. In addition, gravity waves can provide information for warning of the existence of potential hazards (e.g. aircraft turbulence, tsunamis). The lower atmospheric boundary layer involves interacting wind shears, thermal plumes, turbulence, and gravity waves-combining in complex interactions. At times lower level wind shear, instabilities, and gravity waves can produce the erosion of ground based inversions. In other situations thermal plumes can dominate the boundary layer. During the nocturnal boundary layer gravity waves can control the near surface dynamics.

In addition, many major field experiments have measured the properties of atmospheric gravity waves using combinations of remotes sensors and aircraft probes. The DEEPWAVE program (Fritts et al., 2016) is a recent example of an extensive effort to define gravity wave sources and effects in the lower and middle atmosphere. Gravity wave ray trace programs could have a role in helping to guide aircraft and remote sensing measurements. Also, there exist large regional scale efforts to apply high density surface arrays e.g. the USARRAY (Tytell et al., 2016) and also global arrays designed as part of an International Monitoring System (IMS) for monitoring nuclear explosions (Christie and Campus, 2009). Both these arrays have the ability to measure atmospheric gravity wave induced pressure changes. Gravity wave ray tracing can assist in the interpretation of surface array pressure measurements that can provide azimuth and phase speed information.

2.1. Examples of gravity wave generation processes

- Convection (Deardorff et al., 1969; Vadas and Fritts, 2009; Alexander, 1996; Choi et al., 2009; Fovell et al., 1992)
- Severe weather (Bowman and Bedard, 1971; Balachandran, 1980; Curry and Murty, 1974; Nicholls and Pielke, 1994a; b; Taylor and Hapgood, 1988; Dewan et al., 1998; Koch and Siedlarz, 1999)
- Large explosions, volcanoes (ReVelle, 2009)
- Mountain waves (Schoeberl, 1985; Eckermann and Preusse, 1999; Lott and Millet, 2009)
- Terrain forcing (Nappo, 2002)
- Sudden stratospheric warming (Gerrard et al., 2011)
- Fronts (Wrasse et al., 2006; Lin and Zhang, 2008; Plougonven and Zhang, 2014)
- Jets (Suzuki et al., 2013)
- Sea wave forcing (Vadas et al., 2015)
- Geomagnetic activity (Chimonas and Hines, 1970; Testud, 1970)
- Solar eclipse (Chimonas and Hines, 1970)

2.2. Examples of gravity waves in the atmospheric boundary layer

- Gravity waves associated with wind shear induced instabilities (Pramitha et al., 2015)
- Gravity waves associated with temperature inversions (Williams et al., 2002; Bedard et al., 1981; Cunningham and Bedard, 1993)
- Gravity waves associated with the atmospheric boundary layer (Einaudi et al., 1989)
- Gravity waves and boundary layer rolls (Li et al., 2013)
- Gravity waves associated with convective plumes (Vadas and Fritts, 2009)

2.3. Examples of gravity wave initiation of geophysical events

- Severe storm initiation (Uccelini, 1975; Su and Zhai, 2017)
- Sea surface excitation (Šepić et al., 2015)
- Aircraft turbulence (Bedard et al., 1986)
- Modulation of gust surges (Belušić et al., 2004)

2.4. Examples of gravity wave energy transfer

- Atmospheric general circulation models (Lott and Millet, 2009; Alexander et al., 2010; Kim et al., 2003)
- Convectively generated gravity waves (Alexander, 1996)

2.5. Examples of possible gravity wave applications for hazard warning and characterization

- Earthquakes (Mikumo and Watada, 2009)
- Tsunamis (Mikumo and Watada, 2009)
- Meterological Tsunamis (Šepić et al., 2015)
- Earthquake precursors (Lognonné, 2009)
- Tropical Cyclones (Hung and Kuo, 1978; Hung and Smith, 1978; Kim et al., 2009; Hoffmann et al., 2018; Tratt et al., 2018)
- Tornadic storms (Hung et al., 1979)
- Aircraft turbulence (Bedard et al., 1986)

2.6. Examples of gravity waves affecting the middle atmosphere

- (Baumgarten, 2010; Dörnbrack et al., 2017; Gerrard et al., 2004; Krisch et al., 2017)

3. Ray tracing

Ray tracing is a practical method for calculating the ray-theory or WKB approximation for the propagation of waves in a specified medium. Jones (1996, 2007) reviews the conditions for the validity of ray theory and the WKB approximation. Simple tests based on the wavelength of the waves are often inaccurate estimates for the validity of the WKB approximation.

Ray tracing has been used for many years to calculate the propagation of internal gravity waves in the atmosphere (e.g. Eckermann, 1992; Shutts, 1998; Broad, 1999). The propagation of gravity waves in the atmosphere can be calculated using a ray tracing program based on Hamilton's equations (e.g. Slater and Frank, 1947, p. 74) if the dispersion relation is used as the Hamiltonian. The usual barotropic dispersion relation for acoustic-gravity waves (which includes both acoustic and gravity waves) is (Eckart, 1960, eq. (51-2), p. 125) (Gossard and Hooke, 1975, eq. (23-7), p. 112).

$$(\omega_i^2 - 4\Omega_z^2)\left(k_z^2 + \mathbf{k}_A^2 - \frac{\omega_i^2}{C^2}\right) - (k_x^2 + k_y^2)(N^2 - \omega_i^2) = 0\,, \tag{1}$$

where[1]

[1] Equation (2) is consistent with $\mathbf{k}_A \equiv \nabla\rho/(2\rho)$, where ρ is density.