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The effect of anisotropic extra dimension in cosmology

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ABSTRACT

We consider five dimensional conformal gravity theory which describes an anisotropic extra dimension. Reducing the theory to four dimensions yields Brans–Dicke theory with a potential and a hidden parameter α which implements the anisotropy between the four dimensional spacetime and the extra dimension. We find that a range of value of the parameter α can address the current dark energy density compared to the Planck energy density. Constraining the parameter α and the other cosmological model parameters using the recent observational data consisting of the Hubble parameters, type Ia supernovae, and baryon acoustic oscillations, together with the Planck or WMAP 9-year data of the cosmic microwave background radiation, we find $\alpha > -2.05$ for Planck data and $\alpha > -2.09$ for WMAP 9-year data at 95% confidence level. We also obtained constraints on the rate of change of the effective Newtonian constant ($G_{\rm eff}$) at present and the variation of $G_{\rm eff}$ since the epoch of recombination to be consistent with observation.

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1. Introduction

Nowadays, research on the higher dimensional gravity theories like Kaluza–Klein theory, string theory, and brane world scenario constitutes one of the mainstream of theoretical particle physics. In such theories, it is usually taken for granted that the higher dimensional spacetime is isotropic. Even though the isotropic spacetime appeals more aesthetical from the viewpoint of symmetry like Lorentz symmetry and general covariance, this has never been experimentally verified. Therefore, it is a fundamental question to ask whether higher dimensional spacetime has uniform physical properties in all directions [1,2] and envisage the possibility that the extra dimensions might not share the same property with the four dimensional spacetime we are living in.

Recently, an attempt to construct a higher dimensional gravity theory in which the four dimensional spacetime and extra dimensions are not treated on an equal footing was made [3]. It is based on two compatible symmetries of foliation preserving diffeomorphism and anisotropic conformal transformation. The anisotropy is first implemented in the higher dimensional metric by keeping the general covariance only for the four dimensional spacetime. This was achieved by adopting foliation preserving diffeomorphism in which the foliation is adapted along the extra dimensions. Then, it was extended to conformal gravity with introduction of conformal scalar field. In order to realize the anisotropic conformal invariance a real parameter α which measures the degree of anisotropy of conformal transformation between the spacetime and extra dimensional metrics was introduced. In the zero mode effective four dimensional action, it reduces to a scalar-tensor theory coupled with nonlinear sigma model described by extra dimensional metrics. There are no restrictions on the value of α at the classical level. In this paper, we present a cosmological test of the scalar-tensor theory thus obtained in the case of five dimensional theory and check whether or not a specific value of α is preferred.

In general, the conformal invariance constrains the theory in a very tight form in a conformal gravity [4], and contains at most one parameter, that is the potential coefficient λ, $V(\phi) = \frac{\lambda}{4}\phi^4$. The Brans–Dicke theory contains more parameters [5]: one is ω, which is the ratio between the nonminimally coupled $\phi^2 R$ term and kinetic energy term for ϕ. Others are the potential and its respective coefficients, if introduced. It turns out that in the five dimensional anisotropic conformal gravity, the effective four dimensional scalar-tensor theory reduces to the Brans–Dicke theory with a potential, in which the parameter ω and the power of the potential, $V(\phi) \sim \phi^n$, are determined in term of the parameter α. Therefore, from the view point of Brans–Dicke theory, α is a hidden

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parameter and this is a consequence of anisotropic conformal invariance in higher dimensions.

In the gravitational theory with anisotropic conformal invariance, it is more convenient to work with a dimensionless scalar field in order to countercheck the arbitrary anisotropy factor α. Recall that the kinetic coefficient ω of the Brans–Dicke theory can be allowed to be an arbitrary (positive definite) function of the scalar field, $\omega = \omega(\phi)$, which results in a general class of scalar-tensor theories with a dimensionless scalar field and they can be tested with the solar system experiments [5]. In our case, the scalar field is also dimensionless. Nevertheless, ω is constrained to be a constant for the sake of the anisotropic conformal invariance, rendering the theory to be a Brans–Dicke type.

Another important point to be mentioned is that in our four dimensional Brans–Dicke theory, the origin of the Brans–Dicke scalar can be identified with the conformal scalar that is necessarily introduced for the purpose of conformal invariance. It is well-known that in the isotropic case, the conformal or Weyl scalar field is a ghost field with a kinetic coefficient yielding a negative kinetic energy and they cannot become the Brans–Dicke scalar [4]. However, in the anisotropic case, the kinetic coefficient ω is determined as a specific function of α and there exists a range of parameter α where $\omega(\alpha)$ becomes positive. We will check that the actual cosmological test prefers the range of parameter α with a positive value of ω.

The paper is organized as follows: In Section 2, we give a formulation of the 5D gravity with anisotropic conformal invariance and perform dimensional reduction to obtain 4D Brans–Dicke theory. We perform cosmological analysis and give numerical results for evolution equations. In Section 3, comparisons with the recent cosmological data are made and the range of parameter α is constrained. Section 4 contains conclusion and discussion.

2. Model

We start with a formulation of 5D anisotropic conformal gravity. The first part of this section is mostly redrawn from Ref. [3] to make the paper self-contained. Let us first consider the Arnowitt–Deser–Misner (ADM) decomposition of five dimensional metric:

$$ds^2 = g_{\mu\nu}(dx^\mu + N^\mu dy)(dx^\nu + N^\nu dy) + N^2 dy^2. \tag{2.1}$$

Then, the five dimensional Einstein–Hilbert action with cosmological constant is expressed as

$$S_{\rm EH} = \int dyd^4xN\sqrt{-g}\, M_*^3\left[(R - 2\Lambda_5) - \{K_{\mu\nu}K^{\mu\nu} - K^2\}\right], \tag{2.2}$$

where M_* is the five dimensional gravitational constant, R is the spacetime curvature, Λ_5 is the cosmological constant, and $K_{\mu\nu}$ is the extrinsic curvature tensor, $K_{\mu\nu} = (\partial_y g_{\mu\nu} - \nabla_\mu N_\nu - \nabla_\nu N_\mu)/(2N)$. The above action (2.2) can be extended anisotropically by breaking the five dimensional general covariance down to its foliation preserving diffeomorphism symmetry given by

$$x^\mu \to x'^\mu \equiv x'^\mu(x, y), \quad y \to y' \equiv y'(y), \tag{2.3}$$

$$g'_{\mu\nu}(x', y') = \left(\frac{\partial x^\rho}{\partial x'^\mu}\right)\left(\frac{\partial x^\sigma}{\partial x'^\nu}\right) g_{\rho\sigma}(x, y), \tag{2.4}$$

$$N'^\mu(x', y') = \left(\frac{\partial y}{\partial y'}\right)\left[\frac{\partial x'^\mu}{\partial x^\nu}N^\nu(x, y) - \frac{\partial x'^\mu}{\partial y}\right], \tag{2.5}$$

$$N'(x', y') = \left(\frac{\partial y}{\partial y'}\right) N(x, y), \tag{2.6}$$

and non-uniform conformal transformations

$$g_{\mu\nu} \to e^{2\omega(x,y)}g_{\mu\nu}, \;\; N \to e^{\alpha\omega(x,y)}N, \;\; N^\mu \to N^\mu, \;\; \varphi \to e^{-\frac{\alpha+2}{2}\omega}\varphi, \tag{2.7}$$

where a Weyl scalar field φ to compensate the conformal transformation of the metric is introduced. In the above Eq. (2.7), a factor α is introduced in the transformation of $N(=g_{55})$, which characterizes the anisotropy of spacetime and extra dimension.[1] The anisotropic Weyl action invariant under Eqs. (2.3)–(2.7) for an arbitrary α can be written as

$$\begin{aligned} S = \int dyd^4x\sqrt{-g}NM_*^3\Bigg[&\varphi^2\left(R - \frac{12}{\alpha+2}\frac{\nabla_\mu\nabla^\mu\varphi}{\varphi} + \frac{12\alpha}{(\alpha+2)^2}\frac{\nabla_\mu\varphi\nabla^\mu\varphi}{\varphi^2}\right) - \beta_1\varphi^{-\frac{2(\alpha-4)}{\alpha+2}}\left\{B_{\mu\nu}B^{\mu\nu} - \lambda B^2\right\} \\ &+ \beta_2\varphi^2 A_\mu A^\mu - V(\varphi)\Bigg] \end{aligned} \tag{2.8}$$

where β_1, β_2, λ are some constants, the potential V, $B_{\mu\nu}$ and A_μ are given by

$$V = V_0\varphi^{\frac{2(\alpha+4)}{\alpha+2}}, \tag{2.9}$$

$$B_{\mu\nu} = K_{\mu\nu} + \frac{2}{(\alpha+2)N\varphi}g_{\mu\nu}(\partial_y\varphi - \nabla_\rho\varphi N^\rho), \quad B \equiv g^{\mu\nu}B_{\mu\nu}, \tag{2.10}$$

$$A_\mu = \frac{\partial_\mu N}{N} + \frac{2\alpha}{\alpha+2}\frac{\partial_\mu\varphi}{\varphi}. \tag{2.11}$$

A couple of comments are in order. The isotropic case with $\beta_1 = \lambda = \alpha = 1$, and $\beta_2 = 0$ leads to five dimensional Weyl gravity with a potential $V \sim \phi^{\frac{10}{3}}$ [4]. In the anisotropic case, the action (2.8) is, in general, plagued with perturbative ghost instability coming from breaking of the full general covariance of 5D. However, it can be shown that this problem can be cured by constraining the constants β_1 and β_2, especially with $0 < \beta_2 < \frac{3}{2}$ [3].

Now we discuss 4-dimensional effective low energy action and let us consider only zero modes. We first go to a "comoving" frame with $N^\mu = 0$ and impose y-independence (cylindrical condition) for $g_{\mu\nu} = g_{\mu\nu}(x)$, $\phi = \phi(x)$ and $N = N(x)$. This enables to replace $\int dy = L$ where L is the size of the extra dimension and eliminates terms containing $B_{\mu\nu}$ and B. The resulting action preserves the redundant conformal transformation

$$g_{\mu\nu} \to e^{2\omega(x)}g_{\mu\nu}, \quad N \to e^{\alpha\omega(x)}N, \quad \varphi \to e^{-\frac{\alpha+2}{2}\omega(x)}\varphi, \tag{2.12}$$

where $\omega(x, y)$ in (2.7) is replaced with $\omega(x)$. Using this, we further fix $N(x) = 1$ and find the resulting four dimensional action given by

$$S = \int d^4x\sqrt{-g}\left[\frac{\gamma_1 M_p^2}{2}\varphi^2 R - \frac{\gamma_1 M_p^2\omega}{2}\nabla_\mu\varphi\nabla^\mu\varphi - \gamma_2 M_p^4\varphi^{\frac{2\alpha+8}{\alpha+2}}\right], \tag{2.13}$$

where γ_1 and γ_2 are defined as

$$M_*^3 L \equiv \gamma_1 M_p^2/2, \quad M_*^3 L V_0 \equiv \gamma_2 M_p^4, \tag{2.14}$$

and ω is given by

$$\omega \equiv \frac{-4(\alpha+1)(\beta_2\alpha+6)}{(\alpha+2)^2}. \tag{2.15}$$

Let us redefine the field as

$$\varphi \to \tilde{\varphi} = \sqrt{\gamma_1}\varphi.$$

[1] We assume that the field φ is a dimensionless and M_* is a scale related with Planck scale. We also consider only the case $\alpha \neq -2$, because φ is not effected under the conformal transformation in (2.7). It can be actually shown that for $\alpha = -2$, an anisotropic scale invariant gravity theory can be constructed without the need of the field φ.

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