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Superpositions of the cosmological constant allow for singularity resolution and unitary evolution in quantum cosmology

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ABSTRACT

A novel approach to quantization is shown to allow for superpositions of the cosmological constant in isotropic and homogeneous mini-superspace models. Generic solutions featuring such superpositions display unitary evolution and resolution of the classical singularity. Physically well-motivated cosmological solutions are constructed. These particular solutions exhibit characteristic features of a cosmic bounce including universal phenomenology that can be rendered insensitive to Planck-scale physics in a natural manner.

1. Introduction

The 'big bang' singularity and the cosmological constant are well-established features of classical cosmological models [1]. In the context of quantum cosmology, the singularity is typically understood as a pathology that can be expected to be 'resolved' by Planck-scale effects. Most contemporary approaches to resolving the singularity are based upon cosmic bounce scenarios [2]. In contrast, the cosmological constant receives very much the same treatment in classical and quantum cosmological models: it is a constant of nature classically, and thus quantum solutions are supers-selected to eigenstates labelled by its classical value. Cosmological time evolution is unlike either the singularity or the cosmological constant in that the classical and quantum treatments differ. In particular, whereas, the classical treatment of cosmological time is relatively unproblematic, quantum cosmologies based upon standard canonical quantization techniques are described by a 'frozen formalism' that lacks a fundamental evolution equation [3–5]. In this letter, we use a simple model to demonstrate that by treating the cosmological constant differently in quantum cosmological models, one can simultaneously resolve the classical singularity and restore fundamental quantum time evolution. Moreover, a physically well-motivated class of solutions can be constructed that exhibits a cosmic bounce with late-time semi-classical limit peaked on a single value for the cosmological constant.

Three strands of existing research form the basis for our proposal. First, we will appeal to the 'relational quantization' scheme [6–8] that, unlike conventional canonical quantization methods [9–11], is guaranteed to lead to a unitary quantum evolution equation.[1] Second, inspired by other approaches [15,16], we establish generic singularity avoidance in a class of isotropic and homogeneous mini-superspace quantum cosmology models. Third, our model involves superpositions of the cosmological constant in a manner connected to both approach to gravity [17,18] and certain quantum bounce scenarios [19,20].

The model presented here offers physically significant improvements on each of these bodies of existing work. First, we demonstrate that the relational quantization scheme can be applied consistently to a cosmological model and thus provide an exemplar of quantum cosmology with a fundamental unitary evolution equation. Second, the mechanism for singularity avoidance obtained here does not involve the introduction of a Planck-scale cutoff. Rather, observable operators evolve unitarily and remain finite because they are 'protected' by the uncertainty principle. Finally, and

[1] Relational quantization relies upon the observation that the integral curves of the vector field generated by the Hamiltonian constraint in globally reparametrization invariant theories *should not* be understood as representing equivalence classes of physically indistinguishable states since the standard Dirac analysis does not apply to these models [12–14].

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most significantly, we identify an entirely new class of cosmological phenomena that persist into a low energy semi-classical regime. The phenomena in question are rapid 'cosmic beats' with an associated 'bouncing envelope'. The cosmic beats can be identified with Planck-scale effects and, under natural parameter choices, are negligible compared with the effective envelope physics. During the bounce, the envelope behaves in a manner that is closely analogous to *Rayleigh scattering*. The bounce scale due to the effective quantum geometry can thus be made to be relatively large. Significantly, this 'Rayleigh' limit is only available when superpositions of the cosmological constant are allowed and, thus, constitutes a remarkable unique feature of the bouncing unitary cosmologies identified.[2] Finally, it is significant to note that explicit bounce solutions can be shown to exhibit a maximum in the expectation value of the Hubble parameter at some point after the bounce. Furthermore, the additional parameters, which are allowed by permitting superpositions of the cosmological constant, can be seen to slow the rates of change of the effective Hubble parameter in this epoch. This raises the exciting possibility, to be pursued in future work, of describing inflationary scenarios using the model.

2. Model and observables

Consider an homogeneous and isotropic FLRW universe with zero spatial curvature ($k = 0$); scale factor, a; massless free scalar field, ϕ; and cosmological constant, Λ. The field redefinitions

$$v = \sqrt{\frac{2}{3}}a^3 \qquad \varphi = \sqrt{\frac{3\kappa}{2}}\phi\,, \tag{1}$$

where $\kappa = 8\pi G$, give a convenient parameterization of the configuration space in terms of relative spatial volumes, v, and the dimensionless scalar field, φ. The time evolution of the system is given in terms of coordinate time, t, related to the proper time, τ, via the *lapse* function $d\tau = N dt$. The dimensionless lapse, $\tilde{N}$, and cosmological constant, $\tilde{\Lambda}$, can be defined as

$$\tilde{N} = \sqrt{\frac{3}{2}}\frac{\kappa\hbar^2 v N}{V_0} \qquad \tilde{\Lambda} = \frac{V_0^2}{\kappa^2\hbar^2}\Lambda\,, \tag{2}$$

using the reference volume V_0 of some fiducial cell and the (at this point) arbitrary angular momentum scale $\hbar$. In terms of these variables, the mini-superspace Hamiltonian is

$$H = \tilde{N}\left[\frac{1}{2\hbar^2}\left(-\pi_v^2 + \frac{1}{v^2}\pi_\varphi^2\right) + \tilde{\Lambda}\right], \tag{3}$$

where π_v and π_φ are the momenta conjugate to v and φ respectively. In this chart, the Hamiltonian takes the form of a free particle propagating on the upper Rindler wedge, $\mathbb{R}^+_{(1,1)}$ with all non-linearities of gravity appearing in the $1/v^2$ coefficient of the kinetic term for φ. The variables v and ϕ play the role of the usual Rindler coordinates with $v > 0$ playing the role of a time-like (for $\Lambda > 0$) 'radial' coordinate and φ playing the role of a 'boost' variable.

The classical solutions are the geodesics of the upper Rindler wedge. These generically cross the Rindler horizon at $v = 0$, which constitutes the boundary of configuration space. It can be shown, [21], that generic solutions reach $v = 0$ in finite proper time and that the corresponding spacetimes are geodesically incomplete and contain a curvature singularity. This implies a classical singularity in both relevant senses of the Penrose–Hawking singularity theorems.

[2] Two companion papers provide further, more detailed, interpretation and analysis of both general and particular cosmological solutions [21,22].

The Rindler horizon complicates the construction of self-adjoint representations of the operator algebra in the quantum formalism. Consider the Hilbert space, $\mathbb{H} = L^2(\mathbb{R}^+_{(1,1)}, d\theta)$ of square integrable functions on $\mathbb{R}^+_{(1,1)}$ under the Borel measure $d\theta = v\,dv\,d\varphi$. This space is spanned by all functions $(\Phi, \Psi) : \mathbb{R}^+_{(1,1)} \to \mathbb{C}$ satisfying

$$\langle\Phi, \Psi\rangle \equiv \int_{\mathbb{R}^+_{(1,1)}} v\,dv\,d\varphi\,\Phi^\dagger\Psi < \infty. \tag{4}$$

The momentum operator $\hat{\pi}_v$ conjugate to $\hat{v}$ has no self-adjoint extensions because of the restriction $v > 0$. This can be remedied, following [23], by performing the canonical transformation $\mu = \log v$ and $\pi_\mu = v\pi_v$. It is straightforward to show that the symmetric operators

$$\hat{\mu}\Psi = \mu\Psi \qquad \hat{\pi}_\mu = -i\hbar e^{-\mu}\frac{\partial}{\partial\mu}\left(e^\mu\Psi\right) \tag{5}$$

$$\hat{\varphi}\Psi = \varphi\Psi \qquad \hat{\pi}_\varphi = -i\hbar\frac{\partial\Psi}{\partial\varphi} \tag{6}$$

are bounded and essentially self-adjoint and, therefore, form an orthonormal basis for $\mathbb{H}$ according to the spectral theorem. For a geometric approach to this construction, see [21].

3. Unitary quantum cosmology

Application of the quantization scheme presented in [6–8] leads to a Schrödinger-type evolution equation for the system of the form

$$\hat{H}\Psi = i\hbar\partial_t\Psi\,, \tag{7}$$

where the eigenvalues of $\hat{H}$ are to be identified with the (dimensionless) cosmological constant $\tilde{\Lambda}$.[3] The classical Hamiltonian (3) suggests the real and symmetric chart-independent Hamiltonian operator

$$\hat{H} \equiv \frac{1}{2}\Box\,, \tag{8}$$

where $\Box$ is the d'Alambertian operator on Rindler space. The cosmological constant term in (3) is included as a separation constant arising from the general solution to (7) and is interpreted as the negative eigenvalue of $\Box$. An equivalent evolution equation (without the self-adjoint extensions) was presented in [17], motivated by uni-modular gravity approach. This suggests that our proposal may be strongly connected to uni-modular gravity.

A theorem of Von-Neumann [24, X.3] guarantees that self-adjoint extensions of the real, symmetric operator $\hat{H}$ exist. Given an explicit self-adjoint representation of $\hat{H}$, the time evolution is guaranteed to be unitary by Stone's theorem [25, p. 264]. The deficiency subspaces of $\hat{H}$ can be determined by computing its square integral eigenstates for the eigenvalues $\tilde{\Lambda} \to \pm i$. These can be found to be expressible in terms of modified Bessel functions of the second kind (see below) and have rank $(1, 1)$. We therefore expect a $U(1)$ family of self-adjoint extensions, which we parametrize by the log-periodic, positive reference scale Λ_{ref}. To find these extensions explicitly and to construct the general solution to (7), we compute the eigenstates of $\hat{H}$ (with eigenvalues $\tilde{\Lambda}$) in the $v\varphi$-chart. Using the separation Ansatz

[3] For explicit comparison between this equation and the Wheeler–DeWitt type formalism where the right hand side vanishes see [21, II].