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Accelerated expansion of the universe based on emergence of space and thermodynamics of the horizon

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ABSTRACT

Researches in the several decades have shown that the dynamics of gravity is closely related to thermodynamics of the horizon. In this paper, we derive the Friedmann acceleration equation based on the idea of "emergence of space" and thermodynamics of the Hubble horizon whose temperature is obtained from the unified first law of thermodynamics. Then we derive another evolution equation of the universe based on the energy balance relation $\rho V_H = TS$. Combining the two evolution equations and the equation of state of the cosmic matter, we obtain the evolution solutions of the FRW universe. We find that the solutions obtained by us include the solutions obtained in the standard general relativity (GR) theory. Therefore, it is more general to describe the evolution of the universe in the thermodynamic way.

1. Introduction

Numerous astronomical observations tell us that the universe is in accelerated expansion [1–6]. In order to explain the accelerated expansion, a cosmological constant is usually added to the Einstein field equation. In particular, Λ cold dark matter (ΛCDM) model with 4% usual matter, 23% cold dark matter and 73% dark energy describes the accelerated expansion of the universe well. In addition, the accelerated expansion can also been described by modifying the geometrical part of the field equation (for example, $f(R)$ gravity [7,8] and Lanczos–Lovelock gravity).

Recently, Padmanabhan [9,10] have suggested that the difference between the number of the surface degrees of freedom N_{sur} and the number of the bulk degrees of freedom N_{bulk} in a region of space drives the accelerated expansion of the universe through a simple equation $\frac{dV}{dt} = L_p^2(N_{sur} - N_{bulk})$, where V is the Hubble volume and L_p is the Planck length. The standard Friedmann equation of the FRW universe can be derived in this emergence of space scenario. Subsequently, emergent perspective of gravity was further investigated by many researchers [11–13].

On the other hand, it is interesting and meaningful to study cosmology from the point of view of thermodynamics. In 1995, Jacobson [14] argued that the Einstein equation can be derived from the proportionality of entropy and horizon area together with the Clausius relation $\delta Q = TdS$ with δQ and T interpreted as the energy flux and Unruh temperature seen by an accelerated observer just inside the horizon. He also pointed out that the Einstein equation is an equation of state. His paper revealed that thermodynamics of spacetime is closely related to dynamics of gravity. Since then, the relationship between thermodynamics of spacetime and dynamics of gravity have been investigated widely. It has been shown that the field equations are equivalent to the thermodynamic identity $TdS = dE + PdV$ on the horizons in a very wide class of gravitational theories, such as on the static spherically symmetric horizons [15], the stationary axisymmetric horizons and evolving spherically symmetric horizons [16] in Einstein gravity, static spherically symmetric horizons [17] and dynamical apparent horizons [18] in Lanczos–Lovelock gravity, three dimensional BTZ black hole horizons [19,20], etc.

Einstein's equations are equivalent to the unified first law of thermodynamics for the dynamical black hole when the notion of trapping horizon is introduced in the GR theory [21–24]. Inspired by this conclusion, our universe may be considered as a non-stationary gravitational system [25]. Thus we can obtain the temperature of Hubble horizon of the universe based on the unified first law. The advantage of obtaining the horizon temperature in this way is that the temperature has a definite physical origin and a clear mathematical expression.

In many references (e.g. Refs. [11–13,26]) which describe the evolution of the FRW universe, the temperature of Hubble horizon is usually assumed to be $T = H/2\pi$. However, the form of temperature cannot be obtained from an elegant physical principle. In

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this paper, we consider the accelerated expansion of the universe in the late time based on emergence of space and thermodynamics of the Hubble horizon. We employ the temperature obtained from the unified first law of thermodynamics as the temperature of the horizon. Furthermore, we obtain the number of modified bulk degrees of freedom and get the corresponding dynamical equations in the FRW universe based on emergence of space. Then we obtain another evolution equation of the universe based on the energy balance relation $\rho V_H = TS$, where S is the entropy associated with the area of the Hubble sphere and TS is the heat energy of the boundary surface. Combining the two evolution equations and the equation of state of the cosmic matter, we determine the evolution laws of the universe. By analyzing the solutions of the evolution laws, we find the solutions obtained by us include the solutions obtained in the standard general relativity (GR) theory. Therefore, it is more general to describe the evolution of the universe in the thermodynamic way. Next we analyze the solutions in order for the completeness of the discussion. Finally, we make some comments on the alternative perspective for the evolution of the universe.

The present paper is organized as follows. In section 2, we give a simple review about Padmanabhan's work and obtain the temperature of Hubble horizon based on a good motivation. Furthermore, we derive the modified Friedmann equations of the FRW universe based on emergence of space and thermodynamics of the Hubble horizon. In section 3, we obtain the solutions of the Friedmann equations of the FRW universe and analyze their physical meanings. In section 4, some comments on the alternative perspective for the evolution of the universe are made. Finally, we present our conclusions. In this paper, we choose the natural units $c = \hbar = 1$ and the current Hubble constant $H_0 = 70\ \mathrm{km}\cdot\mathrm{s}^{-1}\cdot\mathrm{Mpc}^{-1}$.

2. Modified Friedmann equations

Our universe is homogeneous and isotropic according to astronomical observations and can be described by the FRW metric

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right) = h_{ab}dx^a dx^b + R^2 d\Omega^2, \tag{1}$$

where $R = a(t)r$ is the comoving radius, $h_{ab} = diag\left(-1, \frac{a^2}{1-kr^2}\right)$ is the metric of 2-spacetime ($x^0 = t$, $x^1 = r$) and $k = 0, \pm 1$ denotes the curvature scalar. Solving the Einstein field equation under this metric, we can obtain the properties of the evolution of the FRW universe.

However, Padmanabhan [9,10] came up with a very interesting idea that cosmic space is emergent as cosmic time progresses. First of all, a de Sitter space with the Hubble radius H^{-1} is considered. The temperature of the horizon of such a space is $T = H/2\pi$. He defined the notion of surface and bulk degrees of freedom in a spherical space of radius H^{-1}. The number of the surface degrees of freedom is defined by

$$N_{sur} \equiv \frac{4\pi}{H^2 L_p^2}, \tag{2}$$

where L_p is the Planck length. The number of the bulk degrees of freedom N_{bulk} is given by the equipartition theorem

$$N_{bulk} = \frac{|E|}{(1/2)k_B T} = -\frac{2(\rho+3p)V}{k_B T}, \tag{3}$$

where $|E|$ is the magnitude of Komar energy $|(\rho+3p)|V$, V is Hubble volume $V = 4\pi/3H^3$. The principle of holographic equipartition requires

$$N_{sur} = N_{bulk}. \tag{4}$$

If the equation of state in the de Sitter space is $p = -\rho$, then Eq. (4) can be reduced to $H^2 = 8\pi L_p^2 \rho/3$ which is the standard result of the evolution of the de Sitter universe in the GR theory. This result also shows that the evolution of the de Sitter space obeys the principle of holographic equipartition.

Furthermore, our universe is asymptotically de Sitter, so he thought that the difference between N_{sur} and N_{bulk} drives the universe towards holographic equipartition and the evolution of the universe is dominated by

$$\frac{dV}{dt} = L_p^2 \left(N_{sur} - N_{bulk}\right). \tag{5}$$

Thus substituting Eq. (2) and Eq. (3) into Eq. (5) and using the relations $V = 4\pi/3H^3$, $T = H/2\pi$, one can obtain the following relation

$$\frac{\ddot{a}}{a} = H^2 + \dot{H} = -\frac{4\pi L_p^2}{3}(\rho + 3p). \tag{6}$$

This result is the same as that obtained by solving the Einstein field equation.

In Refs. [27,28], the authors derived the temperature of the horizon of the de Sitter space $T = H/2\pi$ by using the field theory where H^{-1} is the radius of the de Sitter space. However, our universe is asymptotically de Sitter rather than purely de Sitter, so the temperature of the horizon of the universe may not necessarily be expressed as $H/2\pi$. Thus the following two questions arise: What is the expression of the temperature on the horizon of the FRW universe? How can we obtain this expression? Fortunately, we can obtain the expression of the temperature on the horizon of the FRW universe from an elegant physical principle.

The above FRW metric (Eq. (1)) can be written in the double-null form [29,30] as

$$ds^2 = -2d\xi^+ d\xi^- + R^2 d\Omega^2, \tag{7}$$

where $\partial_\pm = \frac{\partial}{\partial \xi^\pm} = -\sqrt{2}\left(\frac{\partial}{\partial t} \pm \frac{\sqrt{1-ar^2}}{a}\frac{\partial}{\partial r}\right)$ are future pointing null vectors.

The trapping horizon is defined as $\partial_+ R|_{R=R_T} = 0$, which gives

$$R_T = \frac{1}{\sqrt{H^2 + \frac{k}{a^2}}} = R_A, \tag{8}$$

where R_A is the apparent horizon. The surface gravity is defined as

$$\kappa = \frac{1}{2\sqrt{-h}}\partial_a\left(\sqrt{-h}h^{ab}\partial_b R\right), \tag{9}$$

so we can get

$$\kappa = -\left(1 + \frac{\dot{H}}{2H^2}\right)H \tag{10}$$

for the apparent horizon of the flat universe. Here we would like to make some explanations on why we choose the flat universe ($k = 0$): (1). Our universe is flat according to astronomical observations; (2). We have chosen $1/H$ as the radius of the horizon when we obtain the dynamical equation of the cosmic evolution, so we also choose Hubble horizon here in order for the consistency.

According to the relation between the surface gravity and the temperature, we obtain the temperature of the horizon [29–31]

$$T = \frac{|\kappa|}{2\pi} = \frac{H}{2\pi}\left(1 + \frac{\dot{H}}{2H^2}\right). \tag{11}$$