Physics Letters B 784 (2018) 373–377

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Accurate nucleon electromagnetic form factors from dispersively improved chiral effective field theory

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ARTICLE INFO

Article history:
Received 13 April 2018
Received in revised form 26 June 2018
Accepted 9 July 2018
Available online 10 August 2018
Editor: W. Haxton

Keywords:
Nucleon form factors
Proton radius puzzle
Dispersion relations
Chiral effective field theory

ABSTRACT

We present a theoretical parametrization of the nucleon electromagnetic form factors (FFs) based on a combination of chiral effective field theory and dispersion analysis. The isovector spectral functions on the two-pion cut are computed using elastic unitarity, chiral pion–nucleon amplitudes, and timelike pion FF data. Higher-mass isovector and isoscalar t-channel states are described by effective poles, whose strength is fixed by sum rules (charges, radii). Excellent agreement with the spacelike proton and neutron FF data is achieved up to $Q^2 \sim 1$ GeV^2. Our parametrization provides proper analyticity and theoretical uncertainty estimates and can be used for low-Q^2 FF studies and proton radius extraction.

1. Introduction

The electromagnetic form factors (EM FFs) parametrize the transition matrix element of the EM current between nucleon states and represent basic characteristics of nucleon structure. The FFs at spacelike momentum transfers $Q^2 \lesssim 1$ GeV^2 have been measured in a series of elastic electron scattering experiments [1–3], most recently at the Mainz Microtron (MAMI) [4–6] and at Jefferson Lab [7–9]. The derivative of the proton electric FF at $Q^2 = 0$ (charge radius) is also determined with high precision in atomic physics experiments. Discrepancies between results obtained with different methods have raised interesting questions concerning the precise value of the proton charge radius and the $Q^2 \to 0$ extrapolation of the elastic scattering data [10–12]. Besides their importance for nucleon structure, the EM FFs are needed as an input in other areas of study, such as precision measurements of quantities used to test the Standard Model.

The experiments and applications require a theoretical description of the FFs that covers a broad range $Q^2 \sim$ few GeV^2 and controls the behavior in the $Q^2 \to 0$ limit (higher derivatives). This can be accomplished using the framework of dispersion theory, which incorporates the analytic properties of the FFs in the momentum transfer. Dispersive parametrizations of the nucleon FFs have been constructed using empirical spectral functions, determined by amplitude analysis techniques and fits to the FF data [13–16]. It would be desirable to have a dispersive parametrization that is based on first-principles dynamical calculations and permits theoretical uncertainty estimates.

In recent work we developed a method for computing the spectral functions of nucleon FFs on the two-pion cut using a combination of chiral effective field theory (χEFT) and amplitude analysis (dispersively improved χEFT, or DIχEFT) [17,18]. The spectral functions are constructed using the elastic unitarity condition. The N/D method is used to separate the $\pi\pi$ rescattering effects (contained in the pion timelike FF) from the coupling of the $\pi\pi$ system to the nucleon (calculable in χEFT with good convergence). The method permits computation of the two-pion spectral functions up to masses $\sim$1 GeV^2 with controlled accuracy. In Ref. [18] the computed spectral functions in LO, NLO, and partial N2LO, accuracy were used to study the FFs at low Q^2 ($<$0.5 GeV^2 for G_E, $<$0.2 GeV^2 for G_M) and their derivatives.

In this letter we use DIχEFT to calculate the nucleon FFs up to $Q^2 \sim 1$ GeV^2 (and higher) and construct a dispersive parametrization of the FFs with theoretical uncertainty estimates. This is achieved by extending our previous calculations in two aspects: (a) We partially include N2LO chiral loop corrections in the isovector magnetic spectral function, by parametrizing them in a form similar to the N2LO corrections in the electric case. This brings the calculation of electric and magnetic isovector FFs up to the same order. (b) We account for higher-mass t-channel states in the spectral functions (isovector and isoscalar) by parametrizing them through effective poles, whose strength is determined by sum rules (charges, magnetic moments, radii). This allows us to extend the

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https://doi.org/10.1016/j.physletb.2018.07.060
0370-2693/

dispersion integrals to higher masses and compute the spacelike FFs up to higher Q^2. We obtain an excellent description of G_E and G_M up to $Q^2 \sim 2$ GeV^2 with controlled theoretical accuracy. Our results represent theoretical predictions in the sense that no fits are performed, and no empirical information from the FFs other than the radii is used in determining the parameters. In the following we describe the calculation and results and discuss potential applications of our FF parametrization.

2. Method

The FFs are analytic functions of the invariant momentum transfer $t \equiv -Q^2$ and satisfy dispersion relations

$$G_i^{p,n}(t) = \frac{1}{\pi}\int_{t_{\rm thr}}^{\infty} dt' \, \frac{\mathrm{Im}\, G_i^{p,n}(t')}{t' - t - i0} \quad (i = E, M). \tag{1}$$

They allow one to reconstruct the spacelike FFs from the spectral functions $\mathrm{Im}\, G_i^{p,n}(t')$ on the cut at $t' > t_{\rm thr}$. For theoretical analysis one uses the isovector and isoscalar combinations, $G_i^{V,S} \equiv \frac{1}{2}(G_i^p \mp G_i^n)$ $(i = E, M)$. In the isovector FF the lowest singularity is the two-pion cut with $t_{\rm thr} = 4M_\pi^2$. The spectral functions on the two-pion cut can be obtained from the elastic unitarity conditions, which in the N/D representation take the form [13,19,20]

$$\mathrm{Im}\, G_E^V(t')[\pi\pi] = \frac{k_{\rm cm}^3}{m_N\sqrt{t'}} \, J_+^1(t') \, |F_\pi(t')|^2, \tag{2}$$

$$\mathrm{Im}\, G_M^V(t')[\pi\pi] = \frac{k_{\rm cm}^3}{\sqrt{2t'}} \, J_-^1(t') \, |F_\pi(t')|^2, \tag{3}$$

where $k_{\rm cm} = \sqrt{t'/4 - M_\pi^2}$ is the center-of-mass momentum of the $\pi\pi$ system in the t-channel. Here $J_\pm^1(t') \equiv f_\pm^1(t')/F_\pi(t')$ are the ratios of the $\pi\pi \to N\bar{N}$ partial-wave amplitudes and the timelike pion FF, which are real for $t' > 4M_\pi^2$ and free of $\pi\pi$ rescattering effects. These functions can be computed in χEFT with good convergence [17,18]. $|F_\pi(t')|^2$ is the squared modulus of the timelike pion FF, which contains the $\pi\pi$ rescattering effects and the ρ meson resonance. This function is measured in $e^+e^- \to \pi^+\pi^-$ exclusive annihilation experiments with high precision and can be taken from a parametrization of the data; see Ref. [21] for a review. Because the $\pi\pi$ state practically exhausts the e^+e^- annihilation cross section at $t' \lesssim 1$ GeV^2, the elastic unitarity relations Eqs. (2) and (3) are assumed to be valid up to $t' = 1$ GeV^2.

The calculation of the $J_\pm^1$ functions in relativistic χEFT is described in Ref. [18]. At LO they are given by the N and Δ Born terms in the $\pi\pi \to N\bar{N}$ amplitudes and the Weinberg–Tomozawa term. At NLO corrections arise at tree-level from an NLO $\pi\pi NN$ contact term in the chiral Lagrangian. At N2LO pion loop corrections appear, and the structure becomes considerably more complex. In Ref. [18] we estimated the N2LO corrections to J_+^1 by assuming that the full N2LO result has the same structure as the tree-level N2LO result, in which the dominant contribution is the term proportional to $d_1 + d_2$. No such estimate was performed for J_-^1, since its N2LO corrections arise entirely from loops. In order to extend the reach of our calculation we now want to estimate J_+^1 and J_-^1 at the same level. This becomes possible with a generalization of our previous arguments. Inspecting the structure of the N2LO loop corrections in the $\pi N \to \pi N$ amplitude, we find that the dominant t-channel correction can be parametrized as

$$A^-[\text{N2LO loop}] = 0, \qquad B^-[\text{N2LO loop}] = \lambda t/f_\pi^2, \tag{4}$$

where A and B are the invariant amplitudes [22]. In this form the N2LO loop result in J_-^1 has the same structure as a tree-level correction arising from contact terms, and the parameter λ can be determined in the same way as in our previous estimate for J_+^1.

In order to extend the isovector spectral integrals to masses $t' > 1$ GeV^2 we need to parametrize the isovector spectral function beyond the two-pion cut. The e^+e^- exclusive annihilation data show that the isovector cross section above $t' \sim 1$ GeV^2 is overwhelmingly in the 4π channel and peaks at $t' \approx 2.3$ GeV^2 [21]. (Incidentally, this value coincides with the squared mass of the ρ' resonance observed in the $\pi\pi$ channel.) It is reasonable to assume that the strength distribution in the nucleon spectral function follows a similar pattern. The simplest way to parametrize the high-mass contribution to the isovector spectral function is by a single effective pole,

$$\mathrm{Im}\, G_{E,M}^V(t')[\text{high-mass}] = \pi a_{E,M}^{(1)} \, \delta(t' - M_1^2), \tag{5}$$

where we choose $M_1^2 = M_{\rho'}^2 = 2.1$ GeV^2. The total isovector spectral function is given by the sum of the $\pi\pi$ cut (calculated in DIχEFT) and the high-mass part (parametrized by the effective pole),

$$\mathrm{Im}\, G_{E,M}^V = \mathrm{Im}\, G_{E,M}^V[\pi\pi] + \mathrm{Im}\, G_{E,M}^V[\text{high-mass}]. \tag{6}$$

We then determine the parameters of the N2LO contributions in $G_{E,M}^V[\pi\pi]$ and the strength of the effective pole in $G_{E,M}^V[\text{high-mass}]$ by imposing the sum rules for the isovector charge and magnetic moment, and for the electric and magnetic radii (here $t_{\rm thr} = 4M_\pi^2$):

$$\frac{1}{\pi}\int_{t_{\rm thr}}^{\infty} dt' \, \frac{\mathrm{Im}\, G_E^V(t')}{t'} = \tfrac{1}{2}, \tag{7}$$

$$\frac{1}{\pi}\int_{t_{\rm thr}}^{\infty} dt' \, \frac{\mathrm{Im}\, G_M^V(t')}{t'} = \tfrac{1}{2}(\mu^p - \mu^n), \tag{8}$$

$$\frac{6}{\pi}\int_{t_{\rm thr}}^{\infty} dt' \, \frac{\mathrm{Im}\, G_E^V(t')}{t'^2} = \langle r^2\rangle_E^V \equiv \tfrac{1}{2}[\langle r^2\rangle_E^p - \langle r^2\rangle_E^n], \tag{9}$$

$$\frac{6}{\pi}\int_{t_{\rm thr}}^{\infty} dt' \, \frac{\mathrm{Im}\, G_M^V(t')}{t'^2} = \langle r^2\rangle_M^V \equiv \tfrac{1}{2}[\mu^p\langle r^2\rangle_M^p - \mu^n\langle r^2\rangle_M^n]. \tag{10}$$

Since the charge and magnetic moment are known precisely, the unknown parameters are essentially determined in terms of the isovector charge and magnetic radii, which can be allowed to vary over a reasonable range (see below). This makes our parametrization particularly convenient for applications where the nucleon radii are regarded as basic parameters or extracted from data.

In the isoscalar FF the lowest singularity is the 3-pion cut ($t_{\rm thr} = 9M_\pi^2$). The strength at $t' < 1$ GeV^2 is overwhelmingly concentrated in the ω resonance, which we describe by a zero-width pole. At $t' \gtrsim 1$ GeV^2 the $K\bar{K}$ and other channels open up. The exclusive e^+e^- annihilation data show that the strength at $t' \sim 1$ GeV^2 is concentrated in the ϕ resonance [21]. We therefore parametrize the high-mass isoscalar strength by an effective pole at the ϕ mass. Altogether, our parametrization of the isoscalar spectral function is

$$\mathrm{Im}\, G_{E,M}^S(t') = \pi a_{E,M}^\omega \delta(t' - M_\omega^2) + \pi a_{E,M}^\phi \delta(t' - M_\phi^2). \tag{11}$$