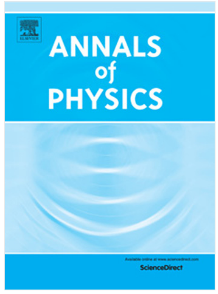

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The Schrödinger Formalism of Electromagnetism and Other Classical Waves — How to Make Quantum-Wave Analogies Rigorous

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Abstract

This paper systematically develops the Schrödinger formalism that is valid also for *gyrotropic* media where the material weights $W = \left(\begin{smallmatrix} \varepsilon & \chi \\ \chi^* & \mu \end{smallmatrix}\right) \neq \overline{W}$ are complex. This is a non-trivial extension of the Schrödinger formalism for *non*-gyrotropic media (where $W = \overline{W}$) that has been known since at least the 1960s [1, 2]. Here, Maxwell's equations are rewritten in the form $\mathrm{i}\partial_t \Psi = M\Psi$ where the selfadjoint (hermitian) *Maxwell operator* $M = W^{-1} \, \mathrm{Rot} \big|_{\omega \geq 0} = M^*$ takes the place of the Hamiltonian and Ψ is a complex wave representing the physical field $(\mathbf{E}, \mathbf{H}) = 2 \mathrm{Re} \, \Psi$. Writing Maxwell's equations in Schrödinger form gives us access to the rich toolbox of techniques initially developed for quantum mechanics and allows us to apply them to classical waves. To show its utility, we explain how to identify conserved quantities in this formalism. Moreover, we sketch how to extend our ideas to other classical waves.

Keywords: Maxwell equations, Maxwell operator, Schrödinger equation, quantum-wave analogies
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Contents

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