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Multimodality in GARCH regression models

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Abstract

It is shown empirically that mixed autoregressive moving average regression models with generalized autoregressive conditional heteroskedasticity (Reg-ARMA-GARCH models) can have multimodality in the likelihood that is caused by a dummy variable in the conditional mean. Maximum likelihood estimates at the local and global modes are investigated and turn out to be qualitatively different, leading to different model-based forecast intervals. In the simpler GARCH(p,q) regression model, we derive analytical conditions for bimodality of the corresponding likelihood. In that case, the likelihood is symmetrical around a local minimum. We propose a solution to avoid this bimodality.

Keywords: ARIMA models; Dummy variable; Forecasting practice; GARCH models; Inflation forecasting; Intervention analysis; Multimodality; Outliers

1. Introduction

Maximum likelihood estimation of GARCH regression models is often considered a routine task. It is not. In this paper we consider one nontrivial aspect of maximum likelihood estimation of GARCH regression models. This aspect is relevant to practitioners who use dummy variables to deal with exceptional observations in (dynamic) regressions. We show that a standard additive outlier treatment can lead to multimodality of the likelihood in dynamic regression models with a GARCH component. Multiple solutions to the likelihood equations lead to different parameter estimates, implying different forecasting behaviour. We derive an analytical expression to explain the source of this problem and provide an easy way to avoid the multimodality.

Dummy variables or pulse variables have been used in linear time series models for a long time, both in intervention analysis, as in Box and Tiao (1975), and in outlier modelling, as in Tsay (1988) and Chen and Liu (1993b). Box and Tiao (1975) discuss likelihood-based inference for intervention effects using different types of dummy variables in autoregressive moving average models. It is well known that neglecting interventions or outliers can have profound effects on model-based forecasts. Ledolter (1989) and Chen and Liu (1993a) discuss the effect of outliers on time series model

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forecasts: "Forecast accuracy in such situations is reduced due to (1) a carry-over effect of the outlier on the point forecast and (2) a bias in the estimates of model parameters" (Chen and Liu, 1993b, p.13). Chen and Liu (1993b) distinguish four outlier types: innovation outliers (IO) and additive outliers (AO), as in Fox (1972), as well as temporary changes and level shifts.

Some of the associated procedures for likelihood-based intervention analysis and outlier detection have been extended to regression models with autoregressive heteroskedasticity, as defined by Engle (1982) and Bollerslev (1986). Franses and Ghijsels (1999) present empirical examples of the relevance of the treatment of additive outliers for diagnostic checking and forecasting with GARCH regression models for stock market volatility. The ARCH (Engle, 1982) and GARCH (Bollerslev, 1986) models are now well-established and widely referenced, see, among many others, Bollerslev, Engle, and Nelson (1994), Gourieroux (1997), and Tsay (2005).

Van Dijk, Franses, and Lucas (1999) discuss the effect of additive outliers on ARCH tests. Carnero, Peña, and Ruiz (2007) study the effect of outliers on GARCH estimates. In GARCH models one can make an additional distinction between additive outliers and volatility outliers, following Hotta and Tsay (1998). In this paper we do not discuss testing for intervention effects or outliers in GARCH regression models. We simply focus on the difficulties of maximum likelihood estimation and the possible consequences for interval forecasting. Based on the results in this paper, we develop an automatic likelihood-based outlier detection procedure for GARCH regression models in a separate study, see Doornik and Ooms (2005).

The main message of our paper is that standard estimates in models involving dummy variables in the conditional means of GARCH regression models have to be treated with great care. In dynamic GARCH regression models, dummy variables for interventions, additive outliers, innovative outliers, or temporary changes, may lead to multimodality and associated problems for statistical inference and forecasting.

In Section 2 we give three examples of empirical models that have multiple modes in the likelihood in the presence of GARCH errors, and show the effect on parameter estimates and forecasts. In Section 3 we analyse the likelihood equations of the GARCH(p,q) model with additive dummies in the regression model, and show analytically why multimodality arises. The analytical expression makes it clear that multimodality is more likely to occur when dummies have their effect before or within volatile periods. Section 4 shows both empirically and analytically that adding a corresponding dummy in the conditional variance equation solves the problem of multimodality.

The results are of practical relevance to the empirical modeller. In a GARCH model with a dummy variable, multimodality may or may not happen. If multimodality does occur:

- there are two solutions for the dummy parameter, with identical standard errors and log-likelihoods, but
- which one is found depends on the starting values and the iterative procedure.
- Unless different starting values are tried, there will be no indication from the output of the computer package whether the 'left' or 'right' estimate was found. This can affect subsequent inference and forecast intervals.
- The local minimum corresponds to what would be expected in a standard regression model: a zero residual for the observation of the dummy.
- As a consequence, starting values from an initial OLS regression (often a default in econometric software), could correspond to the local minimum. Whether the partial derivative is almost zero or just close to zero depends on the precise implementation (and in particular how the GARCH recursion is started). However, if it is zero, the dummy parameter will stay stuck at the local minimum.

Since starting this work, we have had several informal reports of estimation problems in GARCH models with dummies. The results reported below provide both an explanation and a solution. The problem is avoided by adding the lagged dummy to the variance equation; this should become standard empirical practice.

2. Examples of multimodality in GARCH regression models

We present three illustrative examples of multimodality. The first two examples consider inflation series. Earlier versions of these series were used in the