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Forecasting correlated time series with exponential smoothing models

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Abstract

This paper presents the Bayesian analysis of a general multivariate exponential smoothing model that allows us to forecast time series jointly, subject to correlated random disturbances. The general multivariate model, which can be formulated as a seemingly unrelated regression model, includes the previously studied homogeneous multivariate Holt-Winters' model as a special case when all of the univariate series share a common structure. MCMC simulation techniques are required in order to approach the non-analytically tractable posterior distribution of the model parameters. The predictive distribution is then estimated using Monte Carlo integration. A Bayesian model selection criterion is introduced into the forecasting scheme for selecting the most adequate multivariate model for describing the behaviour of the time series under study. The forecasting performance of this procedure is tested using some real examples.

Keywords: Bayesian forecasting; Exponential smoothing; Innovations state space models; Model selection; Monte Carlo methods; Multivariate time series

1. Introduction

Exponential smoothing methods are forecasting techniques which are used widely for the analysis of univariate time series, due to their simplicity and robustness as automatic forecasting procedures (Bermúdez, Segura, & Vercher, 2008; Gardner, 2006; Hyndman, Koehler, Snyder, & Grose, 2002). They originated in the work of Brown and Holt (Brown, 1959; Holt, 1957), but became well known through the paper by Winters (1960). The general form of the exponential smoothing forecast function, involving a set of adaptive coefficients, was given, possibly for the first time, by Box and Jenkins (1976, Appendix A5.3). Snyder (1985) introduced the linear single source of error state space models and showed how they were related to exponential smoothing, while their generalisation to nonlinear state space models was given by Ord, Koehler, and Snyder (1997); see also Koehler, Snyder, and Ord (2001) for a general multiplicative Holt-Winters' model. Without drawing out links with exponential smoothing, single source of error state space models, also known as innovations

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models, had previously been used by Anderson and Moore (1979, pp. 230–238). Innovations state space models provide a general framework where the level, trend and seasonality components of exponential smoothing are stated explicitly in the models (Hyndman, Koehler, Ord, & Snyder, 2008).

For the analysis of dependent time series, that is, series which are subject to correlated random disturbances, or where the observations of a time series are related to the past and present values of other series, the use of multivariate time series models allows information to be borrowed from one series in order to improve the predictions of another series. On such occasions, some multivariate generalisations of the exponential smoothing methods (de Silva, Hyndman, & Snyder, 2007; Enns, Machak, Spivey, & Wrobleski, 1982; Fernández, 1990; Harvey, 1986; Pfeffermann & Allon, 1989) have been shown to provide more satisfactory results than those derived from the univariate analysis of each series. Recently, Bermúdez, Corberán-Vallet, and Vercher (2009a) introduced a new formulation for a multivariate Holt-Winters' model. This model formulation is based on the assumptions that each of the individual time series comes from the univariate Holt-Winters' model, that all of them share a common structure, that is, common smoothing parameters, and that corresponding errors in the univariate models are contemporaneously correlated. Expressing this multivariate Holt-Winters' model as an innovations state space model, the assumption of common smoothing parameters for the univariate models is equivalent to the homogeneity condition assumed in previous studies. This condition implies that all of the individual series have identical time series properties and that the sum of the series also has the same properties (Fernández & Harvey, 1990).

In the case of a common structure for the univariate time series, the estimation of the homogeneous multivariate model is straightforward. Its use both allows the fitting and forecast accuracies to be improved with respect to the univariate model and reduces the computing time required for the analysis of the series considerably (Bermúdez et al., 2009a). However, although univariate series may follow similar processes in some practical situations, this does not necessarily hold in general. In such cases, the use of the homogeneous multivariate model may lead to misleading results. Therefore, it is advisable to use a general multivariate exponential smoothing model where the univariate series are subject to correlated random disturbances but do not necessarily share a common structure.

In this paper, we describe the Bayesian analysis of a general multivariate linear innovations state space model. This general model cannot be formulated as a traditional multivariate regression model, as the homogeneous one can be; instead, it is formulated as a seemingly unrelated regression model (Zellner, 1962), which complicates its analysis. The posterior distribution of all of the unknowns is then obtained from conventional non-informative prior distributions. This posterior distribution is not analytically tractable, but can be approached using MCMC simulation techniques. In particular, we propose a Metropolis-within-Gibbs algorithm that allows us to simulate from the full conditional posterior distributions of the model parameters. The predictive distribution, which encapsulates all of the information concerning the future values of the time series, is finally estimated using Monte Carlo integration.

The paper is organised as follows. In the next section we briefly review linear univariate exponential smoothing, based on innovations state space models, and its multivariate extensions. The Bayesian analysis of the general multivariate model proposed in this paper is developed in Section 3. In Section 4, the Bayesian forecasting procedure, which allows us to obtain point forecasts and prediction intervals, is developed. Section 5 shows the results obtained from the prediction of some correlated time series data sets using our Bayesian forecasting procedure. The last section gives some concluding remarks.

2. Linear exponential smoothing models

2.1. Linear univariate exponential smoothing models

The general linear innovations state space model provides a general framework for linear exponential smoothing. It is defined through the equations (Hyndman, Koehler et al., 2008, p. 34)

$$y_t = w' x_{t-1} + \varepsilon_t \quad \text{(measurement equation)}$$
$$x_t = F x_{t-1} + g \varepsilon_t, \quad \text{(transition equation)}$$

where y_t denotes the observation at time t, and x_t is the state vector which, in exponential smoothing, is the