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Equally weighted portfolios vs value weighted portfolios: Reasons for differing betas☆

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ABSTRACT

We prove that constituent companies' capital structure and tax shield cause the difference in systematic risk between an equally weighted portfolio and a value weighted portfolio in an efficient market where the CAPM holds. The difference in systematic risk has positive association with component companies' default risk.

1. Introduction

We provide an analytical proof explaining why an equally weighted (EW) portfolio has higher return and volatility than a value weighted portfolio (VW) with the same component stocks. Unlike traditional explanations, we assert that the differences exist even in an efficient market where the CAPM holds.

Market efficiency varies across markets and from country to country.[1] It is reasonable to expect that the differences between the EW and VW portfolios diminish in advanced markets or in certain periods if market inefficiency is the major reason for return and volatility difference between the two weighting methods. However, many empirical results report the differences between the two weighting methods, continuously.

The empirical results of Lessard (1976), Ohlson and Rosenberg (1982), Breen et al. (1989), Canina et al. (1998), Fama and French (1992), and Korajczyk and Sadka (2004) suggest that EW portfolios have higher returns than VW portfolios. The results in Lessard (1976), Atchison et al. (1987), Breen et al. (1989), Jegadeesh and Titman (1993), Canina et al. (1998), and DeMiguel et al. (2009) show that the volatility of an EW portfolio is higher than that of a VW portfolio. The results in Whited and Wu (2006) suggest that the market premium of an equally weighted market index is higher than the market premium of a value weighted market index. Furthermore, DeMiguel et al. (2007) report that the EW portfolio outperforms 14 mean–variance portfolio optimization strategies as well as the VW portfolio. EW portfolios tend to have higher CAPM beta then VW portfolio.[2]

There are three schools of research that are relevant to the differences between EW and VW portfolios: noisy market hypothesis, illiquidity, and autocorrelation.

First, the noisy market hypothesis states that the market errors or value tilting cause inflated market capitalizations of overvalued stocks, and thus large capitalization stocks have lower expected returns and undervalued small stocks have higher expected returns according to Arnott (2006). Hsu (2006) proves that the underperformance of VW portfolios compared to other indices is due to the market noise. Arnott et al. (2005) suggest fundamental indexing to avoid this systemic skew and show that fundamental indexing

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[1] For the detail about the recent market efficiency study, refer to Lim and Brooks (2009) and Zhang and Zhang (2013) or http://www.econlib.org/library/Enc/EfficientCapitalMarkets.html

[2] Readers must note that there are exceptions in the empirical results.

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is more mean–variance efficient than VW indices. Arnott et al. (2013) assert that investment strategies often introduce value and small cap tilt to their portfolios. On the contrary, Perold (2007) finds that fundamental indexing is not profitable if investors do not have prior information about the fair values of stocks. Kaplan (2008) also proves the independence theorem of the noisy market hypothesis is not valid.[3]

Second, the higher illiquidity premium of small firms can cause an EW portfolio to have higher return and volatility than a VW portfolio. Amihud (2002) shows that there is a positive return-illiquidity relationship. Blume and Stambaugh (1983) suggest that the bid-ask bounce can inflate volatility of EW portfolios and this effect is mostly due to small and illiquid stocks. They state that the bid-ask effect can cause upward return bias of small size equities and this effect is not large for large size equities. It predicts that the systematic risk difference between EW and VW decreases as the market efficiency improves.

Third, higher autocorrelation of an EW portfolio may cause higher return than a VW portfolio. Roll (1981) relates auto-covariance to size effect. Atchison et al. (1987) predict higher autocorrelation for EW portfolios and the empirical results show the autocorrelation and auto-covariances are higher for EW portfolios compared to VW portfolios. However, higher autocorrelation does not guarantee higher return of a portfolio, and it contradicts that an EW portfolio has higher volatility than a VW portfolio.

We suggest an explanation about the cause of the systematic risk difference between the EW and VW portfolios. We predict that an EW portfolio has higher systematic risk than a VW portfolio in an efficient market where the CAPM holds, and it is because of the tax shield and the capital structure of constituent stocks in each EW and VW portfolio. In addition, our results are not limited by data frequency because our proof does not rely on historical price pattern.

2. Analytical proof

According to Modigliani and Miller (1958), levered equity beta β_i^e increases as a firm issues more debt due to the tax shield. From the MM theorem, when company i is financially healthy with corporate tax τ_i^c, debt D_i, market capitalization E_i, and unlevered equity beta β_i^u, levered equity beta β_i^e satisfies the following equation[4]

$$\beta_i^e = \left[1 + \{1 - \tau_i^c\}\frac{D_i}{E_i}\right] \cdot \beta_i^u. \tag{1}$$

For simplicity, we assume that the corporate tax τ_i^c and unlevered equity beta β_i^u are the same and positive across companies,[5] and the equity expected return r_i follows the CAPM. These assumptions can be relaxed using more realistic equity pricing model. There are criticisms regarding the simplicity of the CAPM, e.g., Fama and French (2004). Due to the model error inherent in the CAPM, our proof does not represent systematic risk difference precisely but rather our proof is valid in the market where the CAPM holds. Our proof provides both upper and lower limit of the systematic risk difference together with a condition when the difference appears. We predict that the difference may not appear when the condition is not satisfied. Likewise, empirical results do not always show that EW portfolios have higher systematic risk than VW portfolios.

Assumption 1. Corporate tax rate τ_i^c and unlevered equity beta β_i^u are the same and positive for all component companies in a portfolio.

Assumption 2. Companies' expected returns follow the CAPM.

We can express the expected return for a portfolio r_p consisting of n assets, assuming market expected return r_m, risk free rate r_f, weight w_i, and expected return r_i of asset i, as

$$r_p = \sum_{i=1}^{n} w_i \cdot r_i = \Phi \cdot \left[\sum_{i=1}^{n} w_i \cdot \frac{D_i}{E_i}\right] + \eta \tag{2}$$

where in the last equality we define the constants Φ and η as

$$\Phi \triangleq (1 - \tau^c)(r_m - r_f)\beta^u$$
$$\eta \triangleq \beta^u(r_m - r_f) + r_f.$$

2.1. Return difference between an EW and a VW portfolio

Eq. (2) is useful for analyzing the return difference between an EW portfolio and a VW portfolio because Φ and η are not affected by the portfolio's weights. Under normal conditions, when the market premium is positive and the corporate tax rate is less than 100%, Φ is a random variable with positive mean. Since the weight of an individual asset in an EW portfolio is $1/n$, the portfolio return r_p^e of an EW portfolio can be expressed as

$$r_p^e = \Phi \cdot \left[\sum_{i=1}^{n} \frac{1}{n} \cdot \frac{D_i}{E_i}\right] + \eta. \tag{3}$$

In a VW portfolio, the weight of asset i is $E_i/\sum_{i=1}^{n} E_i$ at the time of re-balancing. Therefore, we can express a VW portfolio return r_p^v as

$$r_p^v = \Phi \cdot \left[\sum_{i=1}^{n} \frac{E_i}{\sum_{i=1}^{n} E_i} \cdot \frac{D_i}{E_i}\right] + \eta. \tag{4}$$

Thus, the return difference between an EW and a VW portfolio can be expressed as

$$r_p^e - r_p^v = \Phi \cdot \left[\frac{1}{n} \cdot \sum_{i=1}^{n} \frac{D_i}{E_i} - \frac{\sum_{i=1}^{n} D_i}{\sum_{i=1}^{n} E_i}\right]. \tag{5}$$

Eq. (5) implies that the sign of the return difference depends on the difference between the average debt to equity ratio and the ratio of total debt to total equity size in the portfolios when the market premium is positive. It is interesting to note that the return difference disappears under any noise-free correlation between D_i and E_i in Eq. (5). The correlation between debt and equity size is generally positive, so the return difference may diminish as the statistical significance of the correlation strengthens. In reality, it is hard to have a portfolio of which constituent companies' debt size and equity size has a noise-free cross sectional correlation, though. This issue can be explored in a separate paper.

For any portfolio, we can define a range of component companies' debt size in a portfolio such as

$$D - \alpha < D_i < D + \alpha \tag{6}$$

[3] We do not support or decline noisy market hypothesis but suggest them as relevant research.

[4] In the seminal paper of Modigliani and Miller (1958), both debt and equity are defined in terms of market value. Book equity includes assets valued at historical cost. It also includes assets like goodwill that are depreciated for tax purposes but which may effectively grow through time. In particular, representing both Debt and Equity in terms of market value is proper in this research because the beta is used to represent common stock's systematic risk.

[5] We can use upper or lower bounds of unlevered beta, too.