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State and input estimation in phytoplanktonic cultures using quasi-unknown input observers☆

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ABSTRACT

Biological and environmental systems are often influenced by unknown inputs or disturbances, which makes monitoring or state estimation more delicate. In this study, the simultaneous estimation of unmeasured state variables and partly unknown inputs is considered. Only qualitative prior information on these inputs is used in the design procedure, leading to the concept of quasi-unknown input observers (QUIO). These software sensors are applied to the estimation of concentrations, flow rates and light intensity in phytoplanktonic cultures in the chemostat. Implementation and numerical tests are discussed, based on simulation and experimental data.

1. Introduction

State estimation in bioprocesses, by so-called *software sensors*, is particularly important for monitoring and control purposes, as hardware sensors might be expensive (high acquisition and maintenance costs), are not always available for the specific measurement under consideration (due to the difficulty in designing a measurement principle offering accurate and reliable on-line data) and have stringent operating conditions (calibration, processing time, sample destruction in some cases).

Software sensors blend the predictive information of a dynamic process model with the corrective information of available hardware sensors [5,11,9]. However, bioprocess models are usually uncertain due to the inherent difficulty of inferring the model structure (reaction scheme and kinetic laws) and estimating the model parameters (yield coefficients and kinetic parameters) from experimental data. In addition, there exist either disturbances or uncertain input variables (both of them considered as unknown inputs) that affect the performance of the estimators.

Regarding all these possible adverse scenarios, it is therefore required to adopt a robust estimation approach. There are several published results that offer solutions to this problem, for instance, asymptotic observers [1], interval observers [12,15], bundle interval observers [4,16], unknown input observers [7,8,14,18,21,25,26].

In addition to the on-line reconstruction of key-component concentrations, it can also be important to estimate unknown inputs or disturbances acting on the system. It is particularly relevant in environmental processes, such as biological wastewater treatment systems where the influent flow rate and concentrations are often only partially measured, or anaerobic digestion processes where influent concentrations are also partly known. It is also the case in natural ecosystems (such as lakes, rivers, oceans) where external influencing factors are difficult or even impossible to measure.

Unknown input observers (UIO) estimate the state variables of a system robustly with respect to the disturbances or unknown inputs that affect the system. For example, the famous asymptotic observer proposed by Bastin and Dochain in the 90s [1], and which has found so many applications in bioprocess state estimation problems, can be viewed as a specific UIO. The main idea behind this observer is indeed to eliminate the uncertain kinetic model through a state transformation.

The study of UIOs for linear systems is vast [7,8,14,18,26] and several of the proposed design methods take a similar way: finding one (or more) transformations that decouple the effect of the unknown input from a part of the system. However, the existence of such transformations implies severe conditions for the existence of UIOs.

These strong requirements are due to the fact that robustness to *any* disturbance is required, since no prior information on the disturbance is assumed. However, there are cases where some

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qualitative features of the unknown input are indeed known a priori (for example one could know that the disturbance takes the form of a step, but its amplitude remains unknown). In this case, the disturbance is considered as a "quasi-unknown input" and in [19], a design procedure has been proposed, where the known features of the input are defined through an *exosystem*. The benefits of this approach is either to weaken the existence conditions of UIOs or to give more degrees of freedom in the design. The resulting observer is called quasi-unknown input observer (QUIO).

Whereas QUIO have been mostly applied to state estimation and fault detection [22], the focus of the present study is on the simultaneous estimation of unmeasured states and unknown inputs, through a modification of the existence conditions given in [19]. Further, QUIO are evaluated in the framework of monitoring cultures of microalgae operated in chemostat mode. In order to show the practical value of this approach, it is applied to two different case studies, which are investigated both in simulation an with experimental data.

The paper is organized in three main sections. Unknown input observers are introduced first, together with their existence conditions, in Section 2. A decoupling transformation as well as the concept of quasi-unknown inputs are then defined. Finally, the QUIO design methodology is developed. In Section 3, the structure of a dynamic model of microalgae culture is presented, with two particular case studies. Section 4 implements these observers, and evaluate their performance using both numerical simulations and real experimental data. Finally, Section 5 draws some conclusions.

2. Unknown input observers

Consider the following linear system

$$\begin{aligned} \dot{x} &= Ax + Bu + Dw, \qquad x(0) = x_0 \\ y &= Cx, \\ z &= Gx \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^p$ is the known (control) input vector, $y \in \mathbb{R}^m$ is the output (measurement) vector, $w \in \mathbb{R}^q$ is the unknown input vector and $z \in \mathbb{R}^s$ is a linear transformation of the state to be estimated.

An unknown input observer is a dynamical system

$$\begin{aligned} \dot{\xi} &= A_o\xi + B_o[u^T, y^T]^T, \qquad \xi(0) = \xi_0 \\ \hat{z} &= C_o\xi + D_o[u^T, y^T, \dot{y}^T, \ddot{y}^T, \ldots]^T \end{aligned} \tag{2}$$

which produces an estimation of the states of (1) based on the information on measured variables (u and y) and possibly on the derivatives of the output, despite the effect of the unknown input w, that is, $\lim_{t\to\infty}(z - \hat{z}) = 0\,\forall w$.

The state vector of the observer is $\xi \in \mathbb{R}^{n_o}$ and according to its dimension the following classification can be made: (a) if $n_o < n$ the observer is said to be of reduced order; (b) if $n_o = n$ is said to be of full order; (c) if $n_o > n$ is said to be of extended order.

2.1. Existence conditions

There are several approaches to the theory and design of UIOs (see for example Refs. [6,8,13,19,26]), however, there are two necessary and sufficient conditions for the existence of such observers. They are presented in the following lemma.

Lemma 1. [13] *Consider the linear system (1)*

There exists a unknown input state observer ($G = I$) for this system if and only if

$$rk\begin{bmatrix} sI - A & -D \\ C & 0 \end{bmatrix} = n + q, \qquad \forall s \in \mathbb{C}_0^+ \tag{3}$$

$$rk(CD) = rk(D) = q \tag{4}$$

Condition (3) can be understood as a minimum phase condition, since the transmission zeros of the plant cannot be in the right-half closed complex plane ($\mathbb{C}_0^+$), i.e., the matrix in Eq. (3) can be rank deficient for s-values in the left-half complex plane only. This latter interpretation of one of the existence conditions suggests the idea that the system is going to be "inverted", and in order to obtain a "stable" inverted system, the zeros must be in appropriate places.

On the other hand, condition (4) can be understood as a condition of relative degree one for square systems (same number of inputs and outputs). However, from a more general point of view, it implies that the number of outputs (measurements) has to be greater or at least equal to the number of unknown inputs. This is an important and general condition in the framework of UIOs.

2.2. Decoupling transformation

One of the approaches to study UIOs [18] expresses the existence conditions as well as the design of the UIO using a set of transformations that decouple the effect of the unknown inputs and bring the system to a special form. It is formally presented in the next lemma.

Lemma 2. [23,18] *Without loss of generality assume that $rk(D) = q$, $rk(C) = m$, $rk(G) = s$. There exists state $x \to Px$, output $y \to Qy$ and input $u \to Ru$ transformations such that in the new coordinates, system (1) takes the following form*

$$\begin{aligned} \dot{x} &= \begin{bmatrix} A_{11} & 0 & 0 & 0 & A_{15}C_2 \\ A_{21}C_1 & A_{22} & 0 & 0 & A_{25}C_2 \\ A_{31}C_1 & 0 & A_{33} & 0 & A_{35}C_2 \\ A_{41}C_1 & D_2A_{42} & D_2A_{43} & A_{44} & A_{45}C_2 \\ D_1A_{51} & D_1A_{52} & D_1A_{53} & D_1A_{54} & A_{55} \end{bmatrix} x + Bu \\ &\quad + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & D_2 \\ D_1 & 0 \end{bmatrix} w \\ y &= \begin{bmatrix} C_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_2 \end{bmatrix} x \\ z &= \begin{bmatrix} G_1 & G_2 & G_3 & G_4 & G_5 \end{bmatrix} x \end{aligned} \tag{5}$$

and satisfies properties (i)–(vi). Denote $n_i = \dim(A_{ii})$, $i = 1, \ldots, 5$, $q_j = rk(D_j)$, $m_j = rk(C_j)$, $j = 1, 2$.

(i) *Matrices C_1, C_2, D_1 and D_2 are full rank.*
(ii) *Transmission zeros of the system are $\lambda(A_{22}) \cup \lambda(A_{33})$.*
(iii) $\lambda(A_{22}) \subset \mathbb{C}^-$ and $\lambda(A_{33}) \subset \mathbb{C}_0^+$.
(iv) *The couples (C_1, A_{11}) and (C_2, A_{55}) are observable. The couples (A_{44}, D_2) and (A_{55}, D_1) are controllable.*
(v) $q_1 = rk(D_1) = rk(C_2) = m_2$.
(vi) *For every K the couple $(C_2, A_{55} - D_1K)$ is observable, i.e. system (C_2, A_{55}, D_1) is perfectly observable. This implies that*

$$rank\begin{bmatrix} sI - A_{55} & -D_1 \\ C_2 & 0 \end{bmatrix} = n_5 + q_1, \quad \forall s \in \mathbb{C}.$$

A further important consequence of this property is that the state x_5 can be reconstructed from the output $y_2 = C_2x_5$ and a finite number of its derivatives, without information on the input, *i.e. there exist an integer $\delta \geq 0$ and constant matrices M_i such that*

$$x_5 = \sum_{i=0}^{\delta} M_i y_2^{(i)}$$

2.3. Quasi-unknown inputs

In some situations, as discussed in Ref. [19], it may occur that the disturbance that affects a system is not completely unknown, i.e., some qualitative features are known *a priori*, such as its shape or frequency, and thus it is called a “quasi-unknown input”.

The possibility to use these known features has important consequences:

- The existence conditions of an UIO are stringent and the knowledge of some features of the disturbance signals can be used to relax them. In some specific estimation problems, robustness with respect to *all* disturbances is not required, but only with respect to *some* of them. It is then more likely to prove the existence of an observer and to design it.
- The introduction of some information about the disturbances can also give extra degrees of freedom in the assignation of the observer dynamics, as will be shown in the applications.

In this study, where cultures of phytoplankton in the chemostat will be considered, the unknown inputs will be related to flow rates or concentrations that could vary stepwise, or to the light intensity that could vary periodically.

The formal way to introduce the information about the disturbance is by defining an exosystem

$$\begin{aligned} \dot{x}_e &= A_e x_e + D_e v, \qquad x_e(0) = x_{e0} \\ w &= C_e x_e, \\ z_e &= G_e x_e \end{aligned} \tag{6}$$

where $x_e \in \mathbb{R}^{n_e}$ is the state vector, $v \in \mathbb{R}^{q_e}$ is an input vector and $z_e \in \mathbb{R}^{S_e}$ is a linear combination of the states of the exosystem. Matrices A_e, C_e, D_e and G_e are constant of appropriate dimensions.

In the context of this study, the unknown input w in Eq. (6) is produced by an exosystem without inputs, that is $D_e = 0$. To estimate this unknown input, $G_e = C_e$.

- the exosystem that corresponds to a step function, is defined by the following selection of matrices:

$$A_e = 0, \qquad D_e = 0, \qquad C_e = 1. \tag{7}$$

  The amplitude of the step is not part of the information included in the exosystem. The amplitude could be expressed in the initial conditions x_{e0}.
- the exosystem corresponding to a sinusoidal function is defined with

$$A_e = \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix}, \quad D_e = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad C_e = \begin{bmatrix} 1 & 0 \end{bmatrix}. \tag{8}$$

  As can be seen, the only information required about the sinusoidal is its frequency, not its amplitude. Again, the amplitude could be included in the initial conditions.

2.4. Quasi-unknown input observers

With the concept of an exosystem producing the unknown input w, an augmented system can be defined, which includes the plant and the exosystem

$$\begin{aligned} \dot{x}_a &= A_a x_a + B_a u + D_a v, \quad x_a(0) = x_{a0} \\ y &= C_a x_a, \\ z &= G_a x_a, \end{aligned} \tag{9}$$

where

$$x_a^T = [\, x^T, \quad x_e^T \,], A_a = \begin{bmatrix} A & DC_e \\ 0 & A_e \end{bmatrix}, B_a = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

$$D_a = \begin{bmatrix} 0 \\ D_e \end{bmatrix}, C_a = [\, C \quad 0 \,], G_a = \begin{bmatrix} G & 0 \\ 0 & G_e \end{bmatrix}.$$

The design of an unknown input observer for the augmented system (9) implies the design of a quasi-unknown input observer for the plant (1), that is, an observer design which considers a priori knowledge of disturbances (quasi-unknown inputs) [19].

In order to design a QUIO the decoupling transformation will be applied to the augmented system instead of the original system. The following lemma deals with the existence conditions for the QUIO.

Lemma 3. [19] *System (1) has a quasi-unknown input observer, possibly improper, if and only if* $\forall s \in \mathbb{C}_0^+$

$$rk \begin{bmatrix} sI - A_a & -D_a \\ C_a & 0 \end{bmatrix} = rk \begin{bmatrix} sI - A_a & -D_a \\ C_a & 0 \\ G_a & 0 \end{bmatrix}. \tag{10}$$

The observer will have assignable dynamics if and only if (10) is satisfied $\forall s \in \mathbb{C}$.

If $D_e = 0$ then condition (10) is necessary and sufficient for the existence of a strictly proper observer. Its dynamics will be assignable if the condition is satisfied $\forall s \in \mathbb{C}$.

This condition can be simplified in some cases. For example, if the *complete* augmented state vector is going to be estimated, i.e., the process state vector x and the extended state x_e, which is related to the unknown input according to Eq. (6), then $G_a = I$ since $G_e = I$. Under this assumption, condition (10) can be written as follows

$$rk \begin{bmatrix} sI - A & -DC_e \\ 0 & sI - A_e \\ C & 0 \end{bmatrix} = n + n_e \tag{11}$$

3. State and input estimation in phytoplanktonic cultures

The Quasi-unknown input observers are now applied to some monitoring problems in phytoplanktonic cultures. First, dynamic models are described, and a linearization is performed around an operation point. Then the QUIOs are developed step-by-step and tested in simulation and with real data.

3.1. Cultures of phytoplankton in the chemostat

The experimental setup considered in the following is the chemostat, which is a bioreactor operated in continuous mode with identical inlet and outlet flow rates, so as to keep a constant volume. The chemostat is a convenient device to control the growth conditions of phytoplankton and to mimic conditions encountered in natural ecosystems like lakes for instance. The chemostat has been used extensively to study the growth of populations of microorganisms, and competition or cooperation between them [24].

3.2. Dynamic models of phytoplanktonic cultures

One of the most popular models for representing the uptake and growth of phytoplankton on a single substrate is Droop model [10], which has received considerable attention since its publication in 1968. In recent years, a few more detailed models have been proposed, accounting for additional effects, among which the model presented in Ref. [20] is taken as a case study in the present work. This model takes account of the influence of light intensity. Both models can be represented in a generic way as

$$\begin{aligned}\dot{X}(t) &= -D(t)X(t) + \phi_1(P, X)\\ \dot{P}(t) &= -D(t)P + \varphi_1(S, X) + \phi_2(P, X)\\ \dot{S}(t) &= D(t)[S_{in}(t) - S(t)] + \varphi_2(S, X)\end{aligned} \tag{12}$$

where X is a variable related to the biomass concentration in the bioreactor, P is a variable related to the internal substrate pools within the cells, S is related to the external substrate concentration, S_{in} is the inlet substrate concentration to the bioreactor (there is no biomass inlet concentration so that $X_{in} = 0$), $D(t) = F_{in}(t)/V$ represents the dilution rate of the continuous bioreactor with a constant volume V resulting from equal inlet and outlet flow rates $F_{in}(t) = F_{out}(t)$. Functions ϕ_1 and ϕ_2 are related to the growth rate and functions φ_1 and φ_2 represent terms that depend on the uptake rate. In these controlled conditions, decay is not significant and is not accounted for in the models. More details are given in the following.

3.2.1. A simple uptake and growth model: Droop model

The specificity of Droop model [10], as compared to the classical model of Monod [17], is that it uncouples inorganic substrate uptake and growth thanks to an intracellular storage of nutrients, as represented in the following set of equations:

$$\begin{aligned}\dot{X}(t) &= -D(t)X(t) + \bar{\mu}\left(1 - \frac{K_Q}{Q(t)}\right)X(t)\\ \dot{Q}(t) &= \rho_m \frac{S(t)}{S(t)+K_S} - \bar{\mu}\left(1 - \frac{K_Q}{Q(t)}\right)Q(t)\\ \dot{S}(t) &= D(t)[S_{in}(t) - S(t)] - \rho_m \frac{S(t)}{S(t)+K_S}X(t)\end{aligned} \tag{13}$$

In the previous equations, X denotes the biovolume in (μm^3/l) and Q is the internal quota, defined as the quantity of nitrogen per unit of biovolume. The biovolume is the volume of cells in a given volume of culture medium, a quantity representing the biomass concentration, which can be easily and rapidly measured. $D(t)$ is the dilution rate, S is the substrate (inorganic nitrogen) concentration and S_{in} is the input substrate concentration. Function $\rho(S) = \rho_m(S(t)/(S(t) + K_S))$ is the uptake rate, and $\mu(Q) = \bar{\mu}(1 - (K_Q/Q(t)))$ is the growth rate. Parameters K_S and ρ_m represent a half-saturation coefficient for the substrate and the maximum uptake rate, respectively. Parameter $\bar{\mu}$ is the theoretical maximum growth rate, obtained for an infinite internal quota and K_Q is the minimum internal quota allowing growth.

As a particular application, the culture of the chlorophyceae *Dunaliella tertiolecta* is considered. As it is difficult to study the evolution of phytoplankton in the open sea, the growth analysis is carried out in a photo-bioreactor operated in chemostat (equal inflow and outflow resulting in a constant volume). A more complete description of the experimental setup can be found in Ref. [2].

In this study, the objective is to design a software sensor reconstructing Q and S, based on Droop model and on-line measurements of X, in a robust way with respect to unknown variations in the input $w = D(t)$.

For convenience, in a way similar to Ref. [2], a simple change of variable is made in order to simplify the writing of the equations and to avoid the presence of independent terms in the second one,

$$x = T(X, Q, S) = \begin{bmatrix} \dfrac{\rho_m X}{s_i} \\ \dfrac{Q}{K_Q} - 1 \\ \dfrac{S}{s_i} \end{bmatrix}$$

The transformed system reads

$$\begin{aligned}\dot{x} &= f(x) + g(x, u)w\\ y &= x_1 = h(x)\end{aligned} \tag{14}$$

with

$$f(x) = \begin{bmatrix} \dfrac{a_2 x_1 x_2}{x_2 + 1} \\ a_3 \dfrac{x_3}{x_3 + a_1} - a_2 x_2 \\ -\dfrac{x_1 x_3}{x_3 + a_1} \end{bmatrix}, \quad g(x, u) = \begin{bmatrix} -x_1 \\ 0 \\ 1 + u - x_3 \end{bmatrix}$$

where $a_1 = (K_S/s_i) > 0$, $a_2 = \bar{\mu} > 0$, $a_3 = (\rho_m/K_Q) > 0$.

As in Ref. [2], $S_{in}(t) = s_i(1 + u(t))$, $u(t)$ being the input forcing the system ($u(t) > -1$), and s_i the concentration in the influent without input ($u(t) = 0$).

3.2.2. A more detailed model with light influence

In Ref. [20], a more detailed model, taking the influence of the incident light intensity is developed, as

$$\begin{aligned}\dot{X}(t) &= -D(t)X(t) - \lambda X(t) + a(I)L(t)\\ \dot{N}(t) &= -D(t)N(t) + \rho_m \frac{S(t)}{S(t)+K_S}X(t) - \gamma(I)N(t)\frac{L(t)}{X(t)} + \beta L(t)\\ \dot{L}(t) &= -D(t)L(t) + \gamma(I)N(t)\frac{L(t)}{X(t)} - \beta L(t)\\ \dot{S}(t) &= D(t)(S_{in} - S(t)) - \rho_m \frac{S(t)}{S(t)+K_S}X(t)\end{aligned} \tag{15}$$

Variable X now represents the particulate carbon concentration (carbon biomass) and the generic variable P, representing the internal pools, is now a 2-vector with: $P(1) = N$, the internal nitrogen concentration and $P(2) = L$ the chlorophyllian nitrogen concentration. The total particulate nitrogen can be computed as $N + L$.

Functions ϕ_i, $i = 1, 2$ which relate to the growth rate, are affected by the light intensity I, i.e., $\phi_1 = \phi_1(N, L, X, I)$ and $\phi_2 = \phi_2(N, L, X, I)$. β is the coefficient of chlorophyll degradation, λ is the factor of respiration, and functions γ and a describe the influence of the light intensity I in the process

$$\gamma(I) = \frac{\alpha K_L I(t)}{K_I + I(t)} \frac{K_C}{K_C + I(t)}, \quad a(I) = \frac{\alpha I(t)}{K_I + I(t)}$$

As a particular application, the culture of the cryptophyceae *Rhodomonas salina* is considered. The state estimation objective is to reconstruct L, N and S, using the culture model together with on-line measurements of X. The dilution rate D and input concentration S_{in} are known, whereas the light intensity I is the unknown input, which is has to be simultaneously estimated.

In this case, the definition of x, u and w is

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} X \\ L \\ N \\ S \end{bmatrix}, \quad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} D \\ S_{in} \end{bmatrix}, \quad w = I$$

Since this model contains the effect of the light intensity and it is represented by nonlinear terms, the state-space representation of the model can be written as follows

$$\dot{x} = f(x) + g_u(x,u) + g_w(x,w)$$
$$y = h(x) = x_1 \tag{16}$$

where

$$f(x) = \begin{bmatrix} -\lambda x_1 \\ -\beta x_2 \\ \rho_m \dfrac{x_4}{x_4+K_S}x_1 + \beta x_2 \\ -\rho_m \dfrac{x_4}{x_4+K_S}x_1 \end{bmatrix}, \quad g_u(x,u) = \begin{bmatrix} -u_1x_1 \\ -u_1x_2 \\ -u_1x_3 \\ u_1(u_2-x_4) \end{bmatrix},$$

$$g_w(x,w) = \begin{bmatrix} a(w)x_2 \\ \gamma(w)x_3\dfrac{x_2}{x_1} \\ -\gamma(w)x_3\dfrac{x_2}{x_1} \\ 0 \end{bmatrix}$$

In Droop model, whatever input is considered as unknown (S_{in} or D), one can obtain a dynamic equation affine in this input. The second, more detailed, model is again affine in the dilution rate or input concentration, whereas it is not affine with respect to the light intensity.

One possible solution for this structural problem would be to directly consider functions $a(I)$ and $\gamma(I)$ as unknown terms. However, the existence conditions of the QUIO then requires at least two independent measurements.

3.2.3. Model linearization

The considered bioprocess models are nonlinear, but as the cultures are operated in chemostat, i.e., around a stationary point, it is appealing to avoid the use of complex nonlinear state estimation techniques, and to apply the available linear QUIO framework. The next step is therefore a model linearization.

For Droop model (13), considering the steady-state point

$$x = x_L = \begin{bmatrix} x_{1L} \\ x_{2L} \\ x_{3L} \end{bmatrix}, \qquad d = d_L, \qquad u = u_L$$

the linearized system is

$$\dot{\varepsilon}_x = \begin{bmatrix} \gamma_{11} & \gamma_{12} & 0 \\ 0 & \gamma_{22} & \gamma_{23} \\ \gamma_{31} & 0 & \gamma_{33} \end{bmatrix}\varepsilon_x + \begin{bmatrix} 0 \\ 0 \\ \gamma_{34} \end{bmatrix}\varepsilon_u + \begin{bmatrix} \gamma_{15} \\ 0 \\ \gamma_{35} \end{bmatrix}\delta$$
$$\dot{\varepsilon}_x = A_\varepsilon\varepsilon_x + B_\varepsilon\varepsilon_u + D_\varepsilon\delta \tag{17}$$
$$y_L = [\,1 \quad 0 \quad 0\,]\varepsilon_x = C_\varepsilon\varepsilon_x$$

where $\varepsilon_x = x - x_L$, $\varepsilon_u = u - u_L$, $\delta = d - d_L$, and

$$\gamma_{11} = \frac{a_2x_{2L}}{x_{2L}+1} - d_L \qquad \gamma_{31} = -\frac{x_{3L}}{x_{3L}+a_1}$$
$$\gamma_{12} = a_2\frac{x_{1L}}{(x_{2L}+1)^2} \qquad \gamma_{33} = -\frac{a_1x_{1L}}{(x_{3L}+a_1)^2} - d_L$$
$$\gamma_{15} = -x_{1L} \qquad \gamma_{34} = d_L$$
$$\gamma_{22} = -a_2 \qquad \gamma_{35} = 1 + u_L - x_{3L}$$
$$\gamma_{23} = \frac{a_1a_3}{(x_{3L}+a_1)^2}$$

For the model with light influence (15), consider the steady-state point

$$x = x_L = \begin{bmatrix} x_{1L} \\ x_{2L} \\ x_{3L} \\ x_{4L} \end{bmatrix}, u_L = \begin{bmatrix} u_{1L} \\ u_{2L} \end{bmatrix}, w = w_L$$

which can be computed in the following order by:

$$x_{4L} = \frac{K_Su_{1L}[k(w_L)(u_{1L}+\lambda)+(u_{1L}+\beta)]}{\rho_m\gamma(w_L) - u_{1L}[k(w_L)(u_{1L}+\lambda)+(u_{1L}+\beta)]}$$
$$x_{3L} = \frac{(u2L - x_{4L})(u_{1L}+\beta)}{k(w_L)(u_{1L}+\lambda)+(u_{1L}+\beta)}$$
$$x_{1L} = \frac{\gamma(w_L)(u2L - x_{4L})}{k(w_L)(u_{1L}+\lambda)+(u_{1L}+\beta)}$$
$$x_{2L} = \frac{k(w_L)(u2L - x_{4L})(u_{1L}+\lambda)}{k(w_L)(u_{1L}+\lambda)+(u_{1L}+\beta)}$$

where

$$k(w) = \frac{\gamma(w)}{a(w)}$$

The linearized system is

$$\dot{\varepsilon}_x = \begin{bmatrix} \gamma_{11} & \gamma_{12} & 0 & 0 \\ \gamma_{21} & \gamma_{22} & \gamma_{23} & 0 \\ \gamma_{31} & \gamma_{32} & \gamma_{33} & \gamma_{34} \\ \gamma_{41} & 0 & 0 & \gamma_{44} \end{bmatrix}\varepsilon_x + \begin{bmatrix} \eta_{11} & 0 \\ \eta_{21} & 0 \\ \eta_{31} & 0 \\ \eta_{41} & \eta_{42} \end{bmatrix}\varepsilon_u + \begin{bmatrix} \zeta_{11} \\ \zeta_{21} \\ \zeta_{31} \\ 0 \end{bmatrix}\varpi \tag{18}$$
$$\dot{\varepsilon}_x = A_\varepsilon\varepsilon_x + B_\varepsilon\varepsilon_u + D_\varepsilon\varpi$$
$$y_L = [\,1 \quad 0 \quad 0 \quad 0\,]\varepsilon_x = C_\varepsilon\varepsilon_x$$

where $\varepsilon_x = x - x_L$, $\varepsilon_u = u - u_L$, $\varpi = w - w_L$, and

$$\eta_{11} = -x_{1L} \qquad \zeta_{11} = \frac{\alpha K_I}{(K_I+w_L)^2}x_{2L}$$
$$\eta_{21} = -x_{2L}$$
$$\eta_{31} = -x_{3L} \qquad \zeta_{21} = \frac{\alpha K_LK_C(K_IK_C - w_L^2)}{((K_I+w_L)(K_C+w_L))^2}\frac{x_{3L}x_{2L}}{x_{1L}}$$
$$\eta_{41} = u_{2L} - x_{4L}$$
$$\eta_{42} = u_{1L} \qquad \zeta_{31} = -\frac{\alpha K_LK_C(K_IK_C - w_L^2)}{((K_I+w_L)(K_C+w_L))^2}\frac{x_{3L}x_{2L}}{x_{1L}}$$
$$\gamma_{11} = -\lambda - u_{1L}$$
$$\gamma_{12} = \frac{\alpha w_L}{K_I+w_L}$$
$$\gamma_{21} = -\frac{\alpha K_LK_Cw_L}{(K_I+w_L)(K_C+w_L)}\frac{x_{3L}x_{2L}}{x_{1L}^2}$$
$$\gamma_{22} = -\beta - u_{1L} + \frac{\alpha K_LK_Cw_L}{(K_I+w_L)(K_C+w_L)}\frac{x_{3L}}{x_{1L}}$$
$$\gamma_{23} = \frac{\alpha K_LK_Cw_L}{(K_I+w_L)(K_C+w_L)}\frac{x_{2L}}{x_{1L}}$$
$$\gamma_{31} = \rho_m\frac{x_{4L}}{x_{4L}+K_S} + \frac{\alpha K_LK_Cw_L}{(K_I+w_L)(K_C+w_L)}\frac{x_{3L}x_{2L}}{x_{1L}^2}$$
$$\gamma_{32} = \beta - \frac{\alpha K_LK_Cw_L}{(K_I+w_L)(K_C+w_L)}\frac{x_{3L}}{x_{1L}}$$
$$\gamma_{33} = -u_{1L} - \frac{\alpha K_LK_Cw_L}{(K_I+w_L)(K_C+w_L)}\frac{x_{2L}}{x_{1L}}$$
$$\gamma_{34} = \rho_m\frac{x_{1L}K_S}{(x_{4L}+K_S)^2}$$
$$\gamma_{41} = -\frac{\rho_mx_{4L}}{x_{4L}+K_S}$$
$$\gamma_{44} = -\frac{\rho_mx_{1L}K_S}{(x_{4L}+K_S)^2} - u_{1L}$$

4. Design and implementation of the observers

First, the existence conditions are checked for both estimation problems. Then, the design procedure is applied and numerical results are presented and discussed.

4.1. Existence conditions

For Droop model (13), the complete state vector is to be estimated ($G = I$) as well as the unknown input ($G_e = 1$). The existence condition for the augmented system (11) is fulfilled, since

$$rk\begin{bmatrix} sI-A_\varepsilon & -D_\varepsilon C_e \\ 0 & sI-A_e \\ C & 0 \end{bmatrix} = rk\begin{bmatrix} s-\gamma_{11} & -\gamma_{12} & 0 & -\gamma_{15} \\ 0 & s-\gamma_{22} & -\gamma_{23} & 0 \\ -\gamma_{31} & 0 & s-\gamma_{33} & -\gamma_{35} \\ 0 & 0 & 0 & s \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

can be proved to be 4 for all s. This also means that the augmented system has no zero.

For the more detailed model (15), the unknown input is produced by a second order exosystem, so that $G_e = [1\ 0]$ and $G_a \neq I$. However, a condition similar to Eq. (11) can be given

$$rk\begin{bmatrix} sI-A & -DC_e \\ 0 & sI-A_e \\ C & 0 \\ 0 & G_e \end{bmatrix} = n + n_e \tag{19}$$

The existence condition for the augmented system (19) is fulfilled, since

$$rk\begin{bmatrix} sI-A_\varepsilon & -D_\varepsilon C_e \\ 0 & sI-A_e \\ C & 0 \\ 0 & G_e \end{bmatrix} = rk\begin{bmatrix} s-\gamma_{11} & -\gamma_{12} & 0 & 0 & -\zeta_{11} & 0 \\ -\gamma_{21} & s-\gamma_{22} & -\gamma_{23} & 0 & -\zeta_{21} & 0 \\ -\gamma_{31} & -\gamma_{32} & s-\gamma_{33} & \gamma_{34} & -\zeta_{31} & 0 \\ -\gamma_{41} & 0 & 0 & s-\gamma_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & s & -\omega \\ 0 & 0 & 0 & 0 & \omega & s \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

can be proved to be 6 for all s. This also means that the augmented system has no zero.

4.2. Observer design, implementation and tests

A constructive approach using the decomposition of Lemma 2 can now be introduced.

Lemma 4. [18] *Consider system (1) expressed in the special form (5). There exists an UIO (possibly improper) if and only if*

$$G_3 = 0, \quad and \quad G_4 = 0 \tag{20}$$

There exists an UIO with assignable dynamics if and only if

$$G_2 = 0, \quad G_3 = 0, \quad and \quad G_4 = 0 \tag{21}$$

Furthermore, if (20) or (21) are satisfied:

1. *There exists a proper observer if and only if*

$$G_5 = MC_2, \quad \text{for some matrix} \quad M. \tag{22}$$

2. *There exists a strictly proper observer if and only if $G_5 = 0$.*

An observer (possibly of reduced order) with the mentioned properties is given by

$$\dot{\xi}_1 = A_{11}\xi_1 + B_1 u + A_{15}y_2 + H_{11}\left(C_1\xi_1 - y_1\right)$$
$$\dot{\xi}_2 = A_{22}\xi_2 + B_2 u + A_{25}y_2 + A_{21}y_1$$
$$\hat{z} = G_1\xi_1 + G_2\xi_2 + \begin{cases} G_5\sum_{i=0}^{\delta} M_i y_2^{(i)} \\ My_2, \quad if\ (22) \end{cases}$$

where H_{11} is selected such that $(A_{11}+H_{11}C_1)$ is Hurwitz, and M_i is as in Lemma 2, part (vi). If (21) is satisfied, then the corresponding part of ξ_2 is extracted from the observer.

In the following sections we present the application of the design procedure to both models of phytoplankton cultures.

4.2.1. QUIO for Droop model

For Droop model (13) the numerical values of the system parameters are $\rho_m = 9.40$ µmol/mm^3/day, $K_S = 0.105$ µmol/l, $\bar{\mu} = 2$ 1/day, $k_Q = 1.8$ µmol/mm^3, $S_{in} = 100$ µmol/l, $s_i = 100$.

Considering the following steady-state point in the original coordinates $S_L = 0.0479$ µmol/l, $X_L = 30.5409$ mm^3/l), $Q_L = 3.2727$ µmol/mm^3, $d_L = 0.9$ 1/day the steady-state point in the new coordinates is:

$$x_L = \begin{bmatrix} x_{1L} \\ x_{2L} \\ x_{3L} \end{bmatrix} = \begin{bmatrix} 2.8708 \\ 0.8182 \\ 0.0005 \end{bmatrix}$$

The linearized system at this point is

$$\dot{\varepsilon}_x = \begin{bmatrix} 0 & 1.734 & 0 \\ 0 & -2 & 2345 \\ -0.313 & 0 & -1290 \end{bmatrix}\varepsilon_x + \begin{bmatrix} 0 \\ 0 \\ 0.9 \end{bmatrix}\varepsilon_u + \begin{bmatrix} -2.87 \\ 0 \\ 0.999 \end{bmatrix}\delta$$
$$y = [\,1 \quad 0 \quad 0\,]\varepsilon_x = \varepsilon_{x_1}$$

Note that the measured variable in the linearized model is ε_{x_1} and the unknown input is δ.

In this study, no forcing input is considered, i.e., $u = 0$ as well as $u_L = 0$, and only the effect of an unknown dilution rate (disturbance input) is considered.

With these numerical values, conditions (3) and (4) are fulfilled and the existence of an UIO is guaranteed.

Condition (4) is satisfied, since $C_\varepsilon D_\varepsilon = -x_{1L} \neq 0$, and the number of outputs and unknown inputs is the same $m = q = 1$.

The fulfilment of condition (3) can be checked from the results of the transformation (5) applied to the augmented system.

The unknown input (the dilution rate) is considered to be piecewise constant (slowly varying), so the exosystem is chosen as in Eq. (7). When the augmented system is constructed and the decoupling transformation is applied, the following dimensions of the subsystems are obtained $\{n_1 = 4, \quad n_2 = 0, \quad n_3 = 0, \quad n_4 = 0, \quad n_5 = 0\}$. In this case, only one subsystem exists, i.e., subsystem 1, and it has order 4. This means that the inclusion of the exosystem implies that in the new coordinates the whole system is decoupled from the input, and according to property (iv), the couple (C_1, A_{11}) is observable and therefore all 4 poles can be assigned. From another point of view since the augmented system has no transmission zeros, according to Lemma 4 the observer is of assignable dynamics (in the context of Lemma 4 partition G_2 does not exist since $n_2 = 0$).

The selected positions for the observer poles are $[-1, -1290, -5, -1.5]$, which includes two poles in similar positions as the transmissions zeros of the original system.

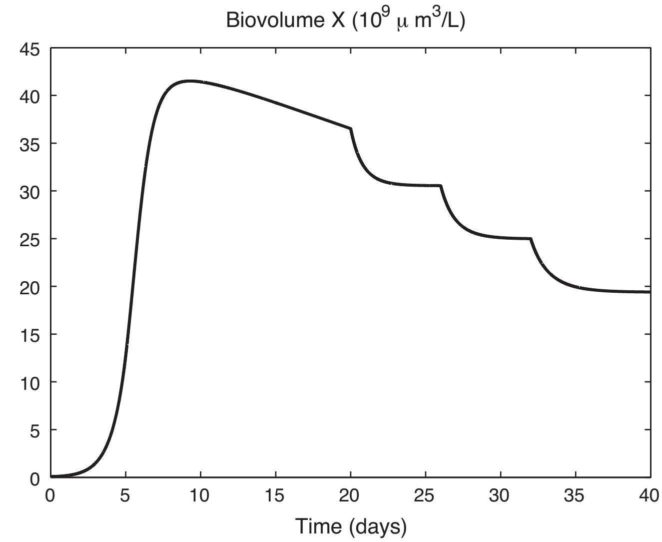

Fig. 1. Simulation of Droop model: evolution of biovolume (measured state variable).

The extended-order observer is then

$$\begin{bmatrix}\dot{\zeta}_1\\ \dot{\zeta}_2\\ \dot{\zeta}_3\\ \dot{\zeta}_4\end{bmatrix} = \begin{bmatrix} -1.3\times 10^3 & 1 & 0 & 0\\ -9.7\times 10^3 & 0 & 1 & 0\\ -18.07\times 10^3 & 0 & 0 & 1\\ -9.7\times 10^3 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\zeta_1\\ \zeta_2\\ \zeta_3\\ \zeta_4\end{bmatrix} + \begin{bmatrix}0\\0\\1.16\\0\end{bmatrix}\varepsilon_u + \begin{bmatrix}0\\2.25\\5.32\\3.07\end{bmatrix} y$$

$$\begin{bmatrix}\hat{z}_1\\ \hat{z}_2\\ \hat{z}_3\\ \hat{\delta}\end{bmatrix} = \begin{bmatrix} 3.15\times 10^3 & 0 & 0 & 0\\ -2.35\times 10^6 & 1.8\times 10^3 & 0 & -1.6\\ 1.29\times 10^6 & -1\times 10^3 & 0.8 & 0\\ 0 & 0 & 0 & -0.95\end{bmatrix}\begin{bmatrix}\zeta_1\\ \zeta_2\\ \zeta_3\\ \zeta_4\end{bmatrix} + \begin{bmatrix}0\\0\\0\\0\end{bmatrix}\varepsilon_u + \begin{bmatrix}0\\0\\0\\0\end{bmatrix} y$$

where the last equation of the observer corresponds to the estimation of the unknown input.

If no exosystem is considered, an unknown input observer can be designed as well. However, it is of non-assignable dynamics and a derivative of the output is needed to estimate the unknown input, which most of the times is not recommended.

4.2.2. QUIO for Droop model: results and discussion

Numerical simulations are first performed to test the observer. Simulation starts with an actual dilution rate linearly increasing with a slope of 0.02 day^{-2} and, after 20 days, the dilution rate experiences three step changes of amplitude 0.2 day^{-1}, at day 20, 26, and 32, respectively. Biovolume, shown in Fig. 1, is measured.

The results are shown in Fig. 2, e.g., the evolution of the estimation error of the two unmeasured variables as well as the estimation of the unknown input. The estimation converges well, showing that not only a stepwise dilution rate, but also a slowly varying dilution rate can be estimated satisfactorily.

The QUIO is now applied to experimental data collected at the Oceanographic Laboratory of Villefranche-sur-Mer, France [2]. The measured biovolume is shown in Fig. 3. The estimation results for the unmeasured variables are shown in Fig. 4. Data points in these figures are not used in the estimation procedure, but for validation purposes only.

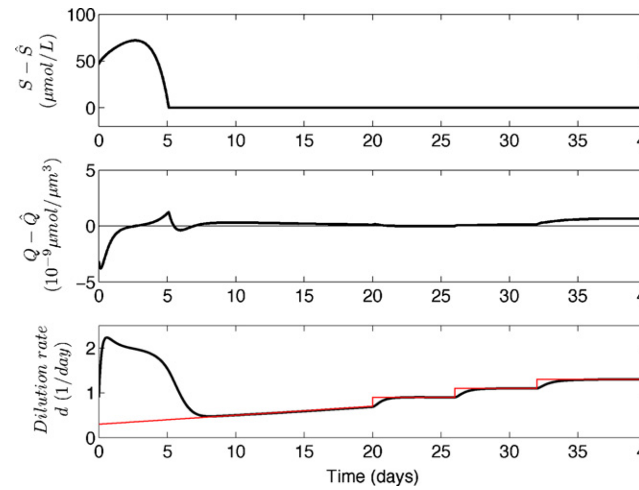

Fig. 2. Simulation of Droop model and application of QUIO. Top: estimation error on S; middle: estimation error on Q; bottom: comparison of dilution rates, real (red line) and estimated (black line). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of the article.)

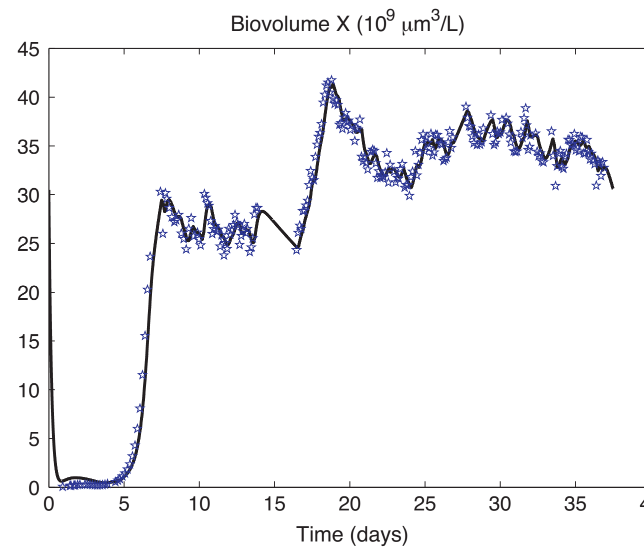

Fig. 3. Experimental evaluation of QUIO based on Droop model: evolution of biovolume (solid lines: estimated variables. Stars: measurements).

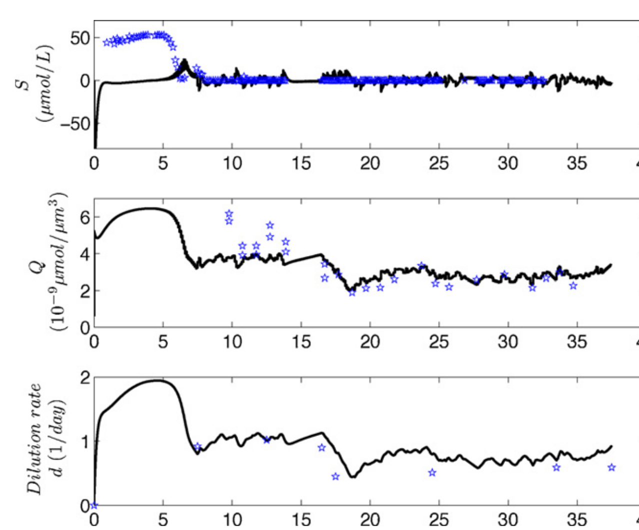

Fig. 4. Experimental evaluation of QUIO based on Droop model (solid lines: estimated variables. Stars: measurements).

As in simulation, the estimation of D is quite satisfactory, with a mean quadratic error of 0.05 day^{-2}.

4.2.3. QUIO for model with light intensity effect

For model (15), the numerical values of the system parameters are $\alpha = 24.1$ day^{-1}, $\beta = 0.345$ day^{-1}, $K_C = 2.8512 \times 10^6$ μE/m^2/day, $K_I = 18.0144 \times 10^6$ μE/m^2/day, $K_L = 6.59$, $K_S = 0.43$ μmol N/l, $\lambda = 0.054$ (day^{-1}), $\rho_m = 0.5$ μmol N/μmol C/day.

Considering the following inputs $u_{1L} = D_L = 0.4$ day^{-1}, $u_{2L} = S_{in} = 10$ μmol N/l, $w_L = I_L = 1.3824 \times 10^6$,μE/m^2/day the operation point is

$$x_L = \begin{bmatrix} x_{1L} \\ x_{2L} \\ x_{3L} \\ x_{4L} \end{bmatrix} = \begin{bmatrix} 27.1365 \\ 7.1726 \\ 2.652 \\ 0.1753 \end{bmatrix}$$

The linearized system at this point is

$$\dot{\varepsilon}_x = \begin{bmatrix} -0.4539 & 1.7176 & 0 & 0 \\ -0.1969 & 0 & 2.0149 & 0 \\ 0.3417 & -0.399 & -2.4149 & 15.9226 \\ -0.1448 & 0 & 0 & -16.3225 \end{bmatrix} \varepsilon_x + \begin{bmatrix} -27.1365 & 0 \\ -7.1726 & 0 \\ -2.652 & 0 \\ 9.8247 & 0.3999 \end{bmatrix} \varepsilon_u + \begin{bmatrix} 0.8277 \\ 0.2328 \\ -0.2328 \\ 0 \end{bmatrix} \times 10^{-5} \varpi$$

$$\varepsilon_y = [\,1 \quad 0 \quad 0 \quad 0\,]\varepsilon_x$$

Note that the measured variable in the linearized model is ε_{x_1} and the unknown input is ϖ.

In this case, small variations in u_{1L} (dilution rate) are considered, but u_{2L} (S_{in}) is constant.

With these numerical values, conditions (3) and (4) are fulfilled and the existence of an UIO is also guaranteed in this case.

Condition (4) is satisfied even in the general case, since $C_\varepsilon D_\varepsilon = (\alpha K_I/(K_I + w_L)^2) x_{2L} \neq 0$, and the number of outputs and unknown inputs is the same $m = q = 1$.

The fulfillment of condition (3) can be also checked from the results of the transformation (5) applied to the augmented system:

In this application, the prior information about the unknown input can be introduced in the design of the observer by an exosystem selected as in Eq. (8). This is an oscillatory system with a fundamental frequency ω, which is assumed to be $\omega = 1$ cycle/day, i.e., it describes in a rough way the periodic evolution of the light during a complete day (i.e. day and night).

With the introduction of the exosystem the decoupling transformation of the augmented system has the following dimensions $\{n_1 = 6, \quad n_2 = 0, \quad n_3 = 0, \quad n_4 = 0, \quad n_5 = 0\}$. That is, with a similar argument as before an observer can be designed with the first subsystem and it has completely assignable dynamics.

The selected positions for the observe poles are $[-1, \quad -2, \quad -3, \quad -14, \quad -38, \quad -10]$.

The extended-order observer is then

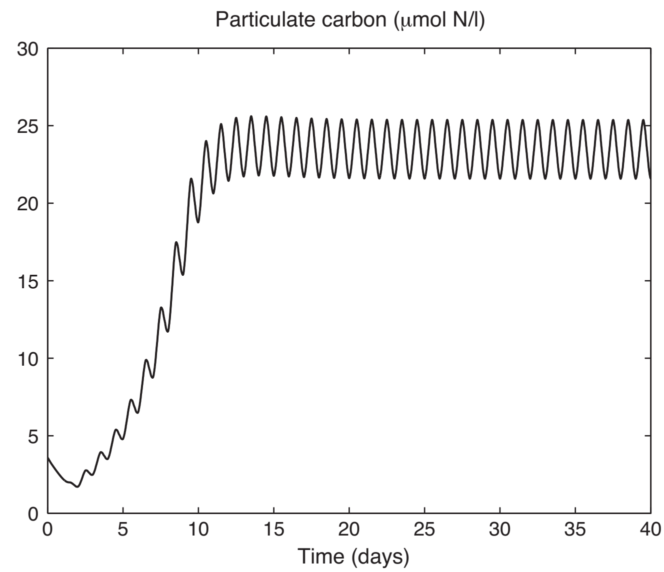

Fig. 5. Simulation of model with light intensity effect: evolution of the particulate carbon (measured variable).

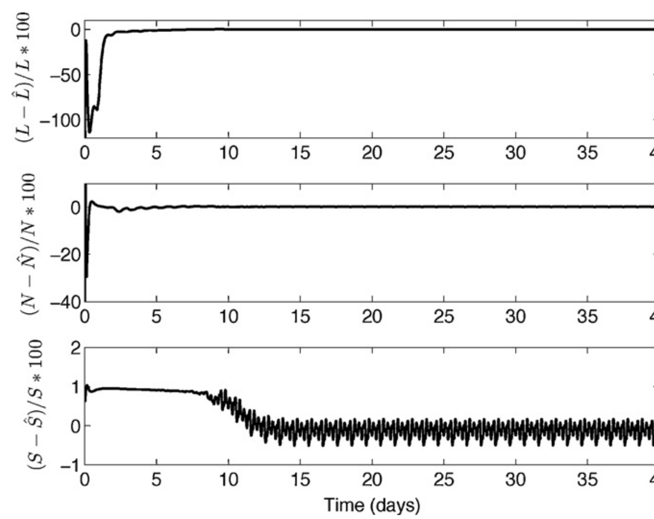

Fig. 6. Simulation of model with light intensity effect and evaluation of QUIO: evolution of estimation errors.

where the last equation of the observer corresponds again to the estimation of the unknown input.

4.2.4. QUIO for model with light intensity effect: results and discussion

In the reference simulation (which mimics reality), light is represented by a fundamental of pulsation $\omega = 1$ cycle/day, and

$$\begin{bmatrix} \dot{\zeta}_1 \\ \dot{\zeta}_2 \\ \dot{\zeta}_3 \\ \dot{\zeta}_4 \\ \dot{\zeta}_5 \\ \dot{\zeta}_6 \end{bmatrix} = \begin{bmatrix} -67.9 & 1 & 0 & 0 & 0 & 0 \\ -1435 & 0 & 1 & 0 & 0 & 0 \\ -1.232 \times 10^4 & 0 & 0 & 1 & 0 & 0 \\ -4.386 \times 10^4 & 0 & 0 & 0 & 1 & 0 \\ -6.483 \times 10^4 & 0 & 0 & 0 & 0 & 1 \\ -3.192 \times 10^4 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \\ \zeta_5 \\ \zeta_6 \end{bmatrix} + \begin{bmatrix} 0.0106 & 0 \\ 0.2026 & 0 \\ 0.9348 & 0 \\ 8.1746 & -0.008 \\ 20.452 & 0 \\ 6.9276 & -0.338 \end{bmatrix} \varepsilon_u + \begin{bmatrix} -0.019 \\ -0.524 \\ -4.484 \\ -16.309 \\ -24.662 \\ -12.296 \end{bmatrix} \varepsilon_y$$

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \hat{x}_4 \\ \hat{\varpi} \end{bmatrix} = \begin{bmatrix} -2.57 \times 10^3 & 0 & 0 & 0 & 0 & 0 \\ 2.49 \times 10^4 & -1.53 \times 10^3 & 79.1 & 1 & -2 & 0 \\ -2.01 \times 10^5 & 1.23 \times 10^4 & -742.1 & 39.5 & 0 & -1 \\ 1.77 \times 10^5 & -1.08 \times 10^4 & 663.1 & -40.6 & 2.5 & -0.1 \\ 6.48 \times 10^8 & 7.7 \times 10^6 & -1.64 \times 10^7 & -1.95 \times 10^5 & 4.16 \times 10^5 & 4.94 \times 10^3 \\ -4.84 \times 10^7 & 1.03 \times 10^8 & 1.22 \times 10^6 & -2.61 \times 10^6 & -3.1 \times 10^4 & 6.62 \times 10^4 \end{bmatrix} \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \\ \zeta_5 \\ \zeta_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \varepsilon_u + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \varepsilon_y$$

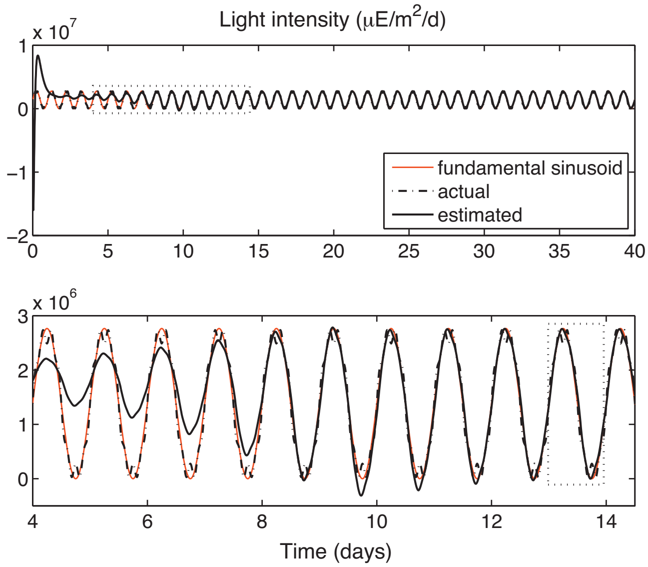

Fig. 7. Simulation of model with light intensity effect and evaluation of QUIO: estimated input.

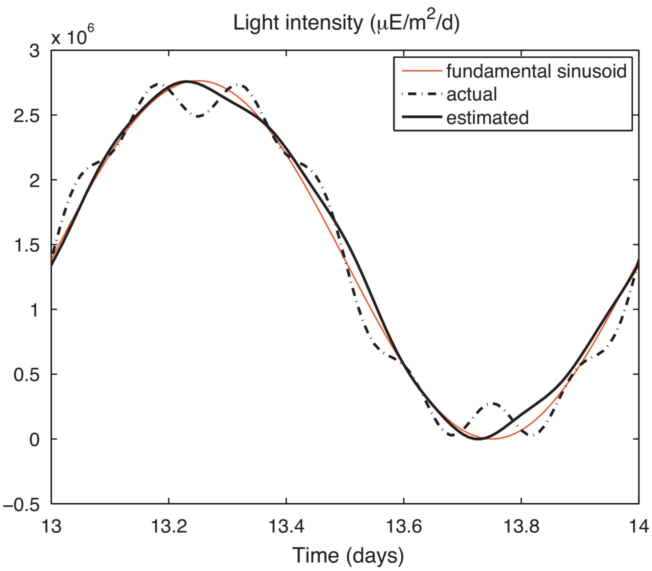

Fig. 8. Simulation of model with light intensity effect and evaluation of QUIO: estimated input (detail).

amplitude between 0 and 2.7648×10^6 $\mu E/m^2/day$. In addition, two harmonics are present, the third and the seventh ones, with an amplitude of 10% of the fundamental.

On the other hand, in the tests, the actual value of the dilution rate $D(t)$ is 10% larger than the value at the steady state point.

The particulate carbon is the measured variable and is shown in Fig. 5 and will be used by both designed observers.

In Fig. 6 (state estimation error) and Fig. 7 (input) the estimation results are shown. The estimation errors tend to zero in spite of the changes in the light intensity. The value of the light intensity used in the linearization is the half of the peak-to-peak amplitude of the sinusoidal, so that the maximal variation is 100% in both directions.

In Fig. 8 a detailed part of Fig. 7 is presented in order to compare the three sinusoidal signals: (1) the fundamental, which is considered in the design; (2) the actual light intensity (fundamental sinusoid plus harmonics); and (3) the estimated unknown input.

The estimated input is close to the fundamental sinusoid.

5. Conclusions

The concept of quasi-unknown inputs, i.e., inputs for which only a few qualitative features would be known a priori, and the introduction of an exosystem defining these features, lead to the design of quasi-unknown input observers, which allow the simultaneous estimation of unmeasured state variables, and disturbances acting on the system. QUIOs are designed for two representative application examples related to the culture of microalgae, and satisfactory results are obtained in simulation and with experimental data. Future work entails new applications, e.g. estimation of influent concentrations in anaerobic digestion.

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