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Transient stabilization of structure preserving power systems with excitation control via energy-shaping

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Abstract

In this paper, the transient stability of multimachine power systems based on structure preserving model (SPM) is considered. The interconnection and damping assignment passivity-based control (IDA-PBC) methodology is extended to solve the excitation regulation problem of SPM represented by a set of differential-algebraic equations. By shaping the total energy function via the introduction of a virtual coupling between the electrical and the mechanical dynamics of the power system, a decentralized excitation control law is proposed to ensure the asymptotic stability of the closed-loop system. The controller is proved to be effective in damping the oscillations and enhancing the system stability by the results of simulation research.

Keywords: Structure preserving model; Passivity-based control; Energy-shaping

1. Introduction

Increasing attention has been devoted to enhancing the stability of power systems, which are characterized by high nonlinearity and influenced by external disturbance. Given the highly nonlinear nature of the power system models, the applicability of linear controller design techniques is severely restricted. Application of nonlinear control method to enhance power system transient stability has attracted much attention. Some achievements using nonlinear control theories including the feedback linearization [1,2] have been accomplished in the past with varying degree of success. These control laws were able to display optimal property, however, the physical meaning is not clear, which has motivated recent works on energy related design techniques [3–8]. The main characteristic of these methods is that the physical structure (Lagrangian or Hamiltonian) is preserved in closed-loop. This has the advantage that the closed-loop energy function can be used as Lyapunov (or storage) function and this render the stability analysis more transparent. Recently, an interconnection and damping assignment passivity-based control (IDA-PBC) methodology has been introduced into the nonlinear control domain of power system [5] and yielded many theoretical extensions and practical applications [9–12]. Analogously to "standard" PBC, the new methodology is based on energy-shaping and passivation principles, but attention is now focused on the interconnection and damping structures of the system. The total energy is shaped via modification of energy transfer between the mechanical and electrical components of the system.

In most of the energy based designs of excitation controller, the methods are employed to the lossless network reduction models represented by a set of ordinary differential equations (ODE). And the synchronous machines are modeled with the classical flux-decay model [13], i.e. quadrature axis synchronous reactance equal to the direct axis transient reactance. While the transmission system itself can be modeled as being lossless without loss of accuracy, the transfer conductance arising from the load impedances is impermissible to be neglected. The classical network reduction of the load buses renders the negligible transfer

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conductances assumption highly unsatisfactory [13,14]. Neglect of transfer conductances leads to a bias in stability estimates of unknown magnitude and direction. Even for the simple swing equation model, the standard energy function of a lossless system cannot be extended to a lossy system [15]. But it is hard to prove that the energy function with the effect of the transfer conductance qualifies as a Lyapunov function. This limitation can be overcome by the use of structure preserving models (SPM) [16–18] which include the full network topology, constant real power and static voltage dependent reactive power loads, higher order generator models. Also these models enjoy a lot of other advantages [19,20].

This article concerns an extension of the IDA control to solve the regulation problem of SPM, which are represented as a set of nonlinear differential-algebraic equations (DAE). The structure of the network is in its original form and the generators are represented by the one-axis model [18]. As usual in IDA designs, a key step in the construction is the modification of the energy transfer between the electrical and the mechanical parts of the system which is obtained via the introduction of state-modulated interconnections. An asymptotically stabilizing law is used to design the excitation controller of structure preserving models.

The remaining of this paper is organized as follows. In Section 2, the modified version of IDA technique for nonlinear differential-algebraic system (NDAS) is presented. This method is applied to the structure preserving power system, and excitation control law is designed to ensure the stability in Section 3. A three-machine simulation study is presented in Section 4 showing the effect on the enhancement of transient stability.

2. Interconnection and damping assignment control of nonlinear differential-algebraic system

Consider the following affine nonlinear differential-algebraic system:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x},\boldsymbol{z}) + \boldsymbol{G}(\boldsymbol{x},\boldsymbol{z})\boldsymbol{u} \\ 0 &= \boldsymbol{\sigma}(\boldsymbol{x},\boldsymbol{z}) \end{aligned} \tag{1}$$

where $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)^{\mathrm{T}} \in R^n$ is the state (differential) vector, $\boldsymbol{z} = (z_1, z_2, \ldots, z_l)^{\mathrm{T}} \in R^l$ is the constraint (algebraic) vector, and $\boldsymbol{u} = (u_1, u_2, \ldots, u_m)^{\mathrm{T}} \in R^m$ is the input vector.

We assume that $\boldsymbol{f}, \boldsymbol{G}, \boldsymbol{\sigma}$ are sufficiently smooth in some open connected set $\boldsymbol{\Omega} \in R^n \times R^l$, and the Jacobian of σ with respect to z has full rank on Ω:

$$\mathrm{rank}(\partial_{\boldsymbol{z}}\boldsymbol{\sigma}(\boldsymbol{x},\boldsymbol{z})) = l, \quad \forall(\boldsymbol{x},\boldsymbol{z}) \in \Omega \tag{A1}$$

where $\partial_z\boldsymbol{\sigma}(\boldsymbol{x},\boldsymbol{z}) = \partial\boldsymbol{\sigma}/\partial z$.

Further, the system possesses equilibrium in $\Omega : \exists(\boldsymbol{x}^*, \boldsymbol{z}^*) \in \boldsymbol{\Omega}$, such that

$$\boldsymbol{f}(\boldsymbol{x}^*, \boldsymbol{z}^*) = 0, \quad \boldsymbol{\sigma}(\boldsymbol{x}^*, \boldsymbol{z}^*) = 0 \tag{A2}$$

Proposition 1. *Suppose (A1) and (A2) hold and if there exist a C^∞ function $H_d(x,z) : R^n \times R^l \to R$ with an isolated minimum at the equilibrium (x^*, z^*), a $n \times n$ skew-symmetric matrix $J_d(x,z) = -J_d(x,z)$, a $n \times n$ semi-positive definite symmetric matrices $\boldsymbol{R}_d(\boldsymbol{x},\boldsymbol{z}) = \boldsymbol{R}_d^{\mathrm{T}}(\boldsymbol{x},\boldsymbol{z}) \geqslant 0$ and two matrix function $A(x,z) = [a_{ij}(x,z)]_{l\times l}$, $B(x,z) = [b_{ij}(x,z)]_{l\times m}$ satisfy the following partial differential equations (PDE) on the open set $\boldsymbol{\Omega}$:*

$$\boldsymbol{G}^{\perp}(\boldsymbol{x},\boldsymbol{z})\boldsymbol{f}(\boldsymbol{x},\boldsymbol{z}) = \boldsymbol{G}^{\perp}(\boldsymbol{x},\boldsymbol{z})[\boldsymbol{J}_d(\boldsymbol{x},\boldsymbol{z}) - \boldsymbol{R}_d(\boldsymbol{x},\boldsymbol{z})]\partial_{\boldsymbol{x}}H_d(\boldsymbol{x},\boldsymbol{z}) \tag{2a}$$

$$\partial_z H_d(\boldsymbol{x},\boldsymbol{z}) = \boldsymbol{A}(\boldsymbol{x},\boldsymbol{z})\boldsymbol{\sigma}(\boldsymbol{x},\boldsymbol{z}) + \boldsymbol{B}(\boldsymbol{x},\boldsymbol{z})\boldsymbol{G}^{\mathrm{T}}(\boldsymbol{x},\boldsymbol{z})\partial_{\boldsymbol{x}}H_d(\boldsymbol{x},\boldsymbol{z}) \tag{2b}$$

where $G^{\perp}(x,z)$ is a left annihilator of $G(x,z)$, i.e. $G^{\perp}(x,z)$ $G(x,z) = 0$. The notation $\partial_x H_d$ is defined by $\partial_x H_d = [\partial_{x1} H_d, \ldots, \partial_{x\,n} H_d]^{\mathrm{T}}$

The closed-loop system (1) will takes the port-controlled Hamiltonian (PCH) form

$$\begin{aligned} \dot{\boldsymbol{x}} &= [\boldsymbol{J}_d(\boldsymbol{x},\boldsymbol{z}) - \boldsymbol{R}_d(\boldsymbol{x},\boldsymbol{z})]\partial_{\boldsymbol{x}}H_d(\boldsymbol{x},\boldsymbol{z}) + \boldsymbol{G}(\boldsymbol{x},\boldsymbol{z})\boldsymbol{u}' \\ 0 &= \boldsymbol{\sigma}(\boldsymbol{x},\boldsymbol{z}) \end{aligned} \tag{3}$$

and be stabilized with control law

$$\mathbf{u}(\boldsymbol{x},\boldsymbol{z}) = \boldsymbol{\alpha}(\boldsymbol{x},\boldsymbol{z}) + \boldsymbol{u}' \tag{4}$$

where

$$\begin{aligned} \boldsymbol{\alpha}(\boldsymbol{x},\boldsymbol{z}) &= [\boldsymbol{G}^{\mathrm{T}}(\boldsymbol{x},\boldsymbol{z})\boldsymbol{G}(\boldsymbol{x},\boldsymbol{z})]^{-1}\boldsymbol{G}^{\mathrm{T}}(\boldsymbol{x},\boldsymbol{z}) \\ &\quad \times \{[\boldsymbol{J}_d(\boldsymbol{x},\boldsymbol{z}) - \boldsymbol{R}_d(\boldsymbol{x},\boldsymbol{z})]\partial_{\boldsymbol{x}}H_d(\boldsymbol{x},\boldsymbol{z}) - \boldsymbol{f}(\boldsymbol{x},\boldsymbol{z})\} \\ \boldsymbol{u}' &= -\mathrm{Sign}[\partial_{\boldsymbol{x}}^{\mathrm{T}}H_d(\boldsymbol{x},\boldsymbol{z})\boldsymbol{G}(\boldsymbol{x},\boldsymbol{z})]\overline{\boldsymbol{B}}^{\mathrm{T}}(\boldsymbol{x},\boldsymbol{z})\bar{\dot{z}} \\ \overline{\boldsymbol{B}} &= [|\boldsymbol{b_{ij}}|]_{\boldsymbol{l\times m}}, \ \bar{\dot{z}} = [|\dot{z}_1|, \ldots, |\dot{z}_l|]^{\mathrm{T}} \end{aligned}$$

$\mathrm{Sign}(\cdot)$ *is a sign function defined as* $\mathrm{Sign}(\boldsymbol{S}) = \begin{bmatrix} \mathrm{sign}(s_1) & & \\ & \ddots & \\ & & \mathit{sign}(s_n) \end{bmatrix}$, $\mathbf{S} = [s_1, \ldots, s_n]$ *and* $\mathrm{sign}(s_i)=1$, *if* $s_i > 0$; $\mathrm{sign}(s_i)=0$, *if* $s_i = 0$; $\mathrm{sign}(s_i) = -1$, *if* $s_i < 0$.

It will be asymptotically stable if, in addition, the largest invariant set contained in

$$\{(\boldsymbol{x},\boldsymbol{z}) \in R^{n\times l} | \partial_{\boldsymbol{x}}^{\mathrm{T}}H_d(\boldsymbol{x},\boldsymbol{z})R_d(\boldsymbol{x},\boldsymbol{z})\partial_{\boldsymbol{x}}H_d = 0, \ \boldsymbol{\sigma}(\boldsymbol{x},\boldsymbol{z}) = 0\} \tag{5}$$

equals $\{(x^*, z^*)\}$.

Proof. Setting up the right hand side of (1), with $u = \alpha(x,z) + u'$, we get the matching equation

$$[\boldsymbol{J}_d(\boldsymbol{x},\boldsymbol{z}) - \boldsymbol{R}_d(\boldsymbol{x},\boldsymbol{z})]\partial_{\boldsymbol{x}}H_d(\boldsymbol{x},\boldsymbol{z}) + \boldsymbol{G}(\boldsymbol{x},\boldsymbol{z})\boldsymbol{u}' = \boldsymbol{f}(\boldsymbol{x},\boldsymbol{z}) + \boldsymbol{G}(\boldsymbol{x},\boldsymbol{z})\boldsymbol{u} \tag{6}$$

Multiplying on the left by $G^{\perp}(x,z)$ we have the PDE (2a).

The expression of $\alpha(x,z)$ is obtained multiplying on the left by the pseudo inverse of $G(x,z)$.

Along the trajectories of (3), we have

$$\begin{aligned} \dot{H}_d(\boldsymbol{x},\boldsymbol{z}) &= -\partial_{\boldsymbol{x}}^{\mathrm{T}}H_d(\boldsymbol{x},\boldsymbol{z})\boldsymbol{R}_d(\boldsymbol{x},\boldsymbol{z})\partial_{\boldsymbol{x}}H_d(\boldsymbol{x},\boldsymbol{z}) \\ &\quad + \partial_{\boldsymbol{x}}^{\mathrm{T}}H_d(\boldsymbol{x},\boldsymbol{z})\boldsymbol{G}(\boldsymbol{x},\boldsymbol{z})(\boldsymbol{u}' + \boldsymbol{B}^T(\boldsymbol{x},\boldsymbol{z})\dot{z}) \\ &\leqslant -\partial_{\boldsymbol{x}}^{\mathrm{T}}H_d(\boldsymbol{x},\boldsymbol{z})\boldsymbol{R}_d(\boldsymbol{x},\boldsymbol{z})\partial_{\boldsymbol{x}}H_d(\boldsymbol{x},\boldsymbol{z}) \end{aligned} \tag{7}$$

Since $R_d(x,z) \geqslant 0$, in accordance with the Lyapunov stability definition, the function $H_d(x,z)$ qualifies as a Lyapunov function for the equilibrium (x^*, z^*). Asymptotic stability follows immediately invoking LaSalle's invariance principle and the condition (5).

The derivative of state variable is often undesired in the control of power system. So if the elements of B and the derivative of state variable z_i are bounded, i.e.

$$|b_{ij}(\boldsymbol{x}, \boldsymbol{z})| \leqslant \phi_{ij}, \quad |\dot{z}_i| \leqslant m_i \quad (i = 1, \ldots, l; \quad j = 1, \ldots, m) \tag{8}$$

the control law can be changed to the form

$$\begin{aligned} \boldsymbol{u}(\boldsymbol{x}, \boldsymbol{z}) = {} & [\boldsymbol{G}^{\mathrm{T}}(\boldsymbol{x}, \boldsymbol{z})\boldsymbol{G}(\boldsymbol{x}, \boldsymbol{z})]^{-1}\boldsymbol{G}^{\mathrm{T}}(\boldsymbol{x}, \boldsymbol{z})\{[\boldsymbol{J}_d(\boldsymbol{x}, \boldsymbol{z}) \\ & - \boldsymbol{R}_d(\boldsymbol{x}, \boldsymbol{z})]\partial_x H_d(\boldsymbol{x}, \boldsymbol{z}) - \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{z})\} \\ & - \mathrm{Sign}[\partial_{\boldsymbol{x}}^{\mathrm{T}} H(\boldsymbol{x}, \boldsymbol{z})\boldsymbol{G}(\boldsymbol{x}, \boldsymbol{z})]\Phi^{\mathrm{T}}\mathbf{m} \end{aligned} \tag{9}$$

where $\boldsymbol{\Phi} = [\phi_{ij}]_{l\times m}$, $\mathbf{m} = [m_1, \ldots, m_l]^{\mathrm{T}}$.

$J_d(x,z)$and $R_d(x,z)$, which represent the desired interconnection structure and dissipation, respectively. They are selected by the designer. $H_d(x,z)$, the desired total stored energy, may be totally, or partially, fixed provided we can ensure the minimum at the equilibrium. □

3. Excitation controller design of structure preserving model

In this section, we consider the problem of transient stability of a large-scale power system. The result presented in Section 2 is applied to design the excitation controllers.

A power system with n machines, $n+m+1$ buses and nonlinear voltage dependent loads is studied [7]. All generators are represented by the one-axis E'_q model. The mechanical torque is assumed to be constant. The resistance of the transmission lines is eliminated. Buses from 1 to n are the terminal buses of the generators. Bus $n+1$ is an infinite bus. Bus from $n+2$ to $n+m+1$ are the load buses. The node admittance matrix is $\boldsymbol{Y} = [Y_{ij}] = [jB_{ij}]$, where B_{ij} is the susceptance of the line connecting bus i and j. The voltage phasor of the ith Bus is expressed as, $V_i\angle\theta_i$. All phase angles are measured relative to the infinite bus.

The real and reactive power demand at the ith load bus are P_i^d and Q_i^d. The real power is represented as a constant, and the reactive power is depended on the voltage of the bus, i.e. $Q_i^d = Q_i^d(V_i)$.

The dynamic of ith machine including flux-decay is described by the one-axis model

$$\begin{aligned} \dot{\delta}_i &= \omega_i - \omega_0 \\ M_i\dot{\omega}_i &= \omega_0\left[P_{mi} - P_{ei} - \frac{D_i}{\omega_0}(\omega_i - \omega_0)\right] \\ \frac{T'_{d0i}}{x_{di} - x'_{di}}\dot{E}'_{qi} &= K_i(\delta_i, E'_{qi}, \theta_i, V_i) + \frac{1}{x_{di} - x'_{di}}E_{fi}, \quad i = 1, \ldots, n \end{aligned} \tag{10}$$

where

$$P_{ei} = \frac{x'_{di} - x_{qi}}{2x_{qi}x'_{di}}V_i^2\sin(2(\delta_i - \theta_i)) + \frac{1}{x'_{di}}E'_{qi}V_i\sin(\delta_i - \theta_i)$$

$$K_i(\delta_i, E'_{qi}, \theta_i, V_i) = -\frac{x_{di}}{x'_{di}(x_{di} - x'_{di})}E'_{qi} + \frac{1}{x'_{di}}V_i\cos(\delta_i - \theta_i)$$

δ_i is rotor angle, ω_i the rotor angle speed, $\omega_0 = 2\pi f_0$ the synchronous machine speed, f_0 the synchronous frequency, D_i the damping constant, M_i the inertia constant, x_{di}, x_{qi} the direct and quadrature axis synchronous reactance, x'_{di} the direct axis transient reactance, E'_{qi} the quadrature axis voltage behind transient reactance, P_{mi} the mechanical power, T'_{d0i} the direct axis transient open-circuit time constant, E_{fi} the input of the exciter.

At the ith generator terminal buses, $i = 1, \ldots, n$, we get

$$\begin{aligned} 0 = g_i = {} & \frac{x'_{di} - x_{qi}}{2x'_{di}x_{qi}}V_i^2\sin(2(\theta_i - \delta_i)) \\ & + \frac{1}{x'_{di}}E'_{qi}V_i\sin(\theta_i - \delta_i) + \sum_{j\neq i}^{n+m+1} V_iV_jB_{ij}\sin(\theta_i - \theta_j) \end{aligned} \tag{11}$$

$$\begin{aligned} 0 = h_i = V_i^{-1}\Bigg[& \frac{x'_{di} + x_{qi}}{2x'_{di}x_{qi}}V_i^2 - \frac{1}{x'_{di}}E'_{qi}V_i\cos(\theta_i - \delta_i). \\ & - \frac{x'_{di} - x_{qi}}{2x'_{di}x_{qi}}V_i^2\cos(2(\theta_i - \delta_i)) - B_{ii}V_i^2 \\ & - \sum_{j\neq i}^{n+m+1} V_iV_jB_{ij}\cos(\theta_i - \theta_j)\Bigg] \end{aligned} \tag{12}$$

At the ith load terminal buses, $i = n+2, \ldots, n+m+1$, we get

$$0 = g_i = \sum_{j\neq i}^{n+m+1} V_iV_jB_{ij}\sin(\theta_i - \theta_j) + P_i^d \tag{13}$$

$$0 = h_i = V_i^{-1}\left[-B_{ii}V_i^2 - \sum_{j\neq i}^{n+m+1} V_iV_jB_{ij}\cos(\theta_i - \theta_j) + Q_i^d(V_i)\right] \tag{14}$$

The system under study can be described as following compact vector form:

$$\begin{aligned} \dot{\boldsymbol{\delta}} &= \boldsymbol{\omega} - \boldsymbol{\omega}_0 \\ \boldsymbol{M}\dot{\boldsymbol{\omega}} &= \omega_0\left[\boldsymbol{P}_m - \boldsymbol{P}_e - \frac{\boldsymbol{D}}{\omega_0}(\omega - \omega_0)\right] \\ \frac{T'_{d0}}{x_d - x'_d}\dot{\boldsymbol{E}}'_q &= \boldsymbol{K}(\boldsymbol{\delta}, \boldsymbol{E}'_{\boldsymbol{q}}, \boldsymbol{\theta}, \boldsymbol{V}) + \frac{1}{x_d - x'_d}\boldsymbol{E}_f \\ 0 &= \boldsymbol{g}(\boldsymbol{\delta}, \boldsymbol{E}'_{\mathbf{q}}, \boldsymbol{\theta}, \boldsymbol{V}) \\ 0 &= \boldsymbol{h}(\boldsymbol{\delta}, E'_{\boldsymbol{q}}, \boldsymbol{\theta}, \boldsymbol{V}) \end{aligned} \tag{15}$$

The choice of the desired matrices $\boldsymbol{J}_d$ and $\boldsymbol{R}_d$ is based on physical considerations. Injecting additional damping into the electrical variable E'_{qi} is easily achieved feeding back $\partial H/\partial E'_{qi}$ [7]. On the other hand, it can be seen that the damping in the mechanical coordinates (δ_i, ω_i) is weak, since $\boldsymbol{D}$ is usually very small. Furthermore, if the interconnection matrix $\boldsymbol{J}_d$ does not contain any coupling between the electrical and the mechanical dynamics, the propagation of the damping injected in the mechanical coordinates is far from obvious. This suggests the new interconnection and damping matrices

$$J_i = \begin{bmatrix} 0 & \frac{\omega_0}{M_i} & 0 \\ -\frac{\omega_0}{M_i} & 0 & J_{i1} \\ 0 & -J_{i1} & 0 \end{bmatrix}, \quad R_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{\omega_0 D_i}{M_i^2} & 0 \\ 0 & 0 & k_{ei} \end{bmatrix} \tag{16}$$

where J_{i1} is unknown and will be fixed by solve the Eq. (2a).

Here the energy function is given

$$H_d(x,z) = H_1(x,z) + H_2(x,z) \tag{17}$$

where $H_1 = \sum_{i=1}^{n} \frac{\gamma_i}{2}\left[E'_{qi} - E'^{*}_{qi} + k_{i1}[V_i\cos(\delta_i - \theta_i) - V_i^*\cos(\delta_i^* - \theta_i^*)]\right]^2$,

$$\begin{aligned} H_2 = {} & \sum_{i=1}^{n} \frac{M_i}{2\omega_0}(\omega_i - \omega_0)^2 + \sum_{i=1}^{n} \frac{1}{2x'_{di}k_{i1}}\left(E'_{qi} - E'^{*}_{qi}\right)^2 \\ & - \sum_{i=1}^{n} P_{mi}(\delta_i - \delta_i^*) - \sum_{i=1}^{n} \frac{(x'_{di} - x_{qi})}{4x'_{di}x_{qi}}[V_i^2\cos(2(\delta_i - \theta_i)) \\ & - V_i^{*2}\cos(2(\delta_i^* - \theta_i^*))] + \sum_{i=1}^{n} \frac{x'_{di} + x_{qi}}{4x'_{di}x_{qi}}(V_i^2 - V_i^{*2}) \\ & - \sum_{i=1}^{n} \frac{E'^{*}_{qi}}{x'_{di}}[V_i\cos(\delta_i - \theta_i) - V_i^*\cos(\delta_i^* - \theta_i^*)] \\ & + \sum_{k=n+2}^{n+m+1} P_k^d(\theta_k - \theta_k^*) + \sum_{k=n+2}^{n+m+1} \int_{V_k^*}^{V_k} \frac{1}{V} Q_k^d(V)dV \\ & - \frac{1}{2}\sum_{i=1, i\neq n+1}^{n+m+1} \sum_{j=1}^{n+m+1} \Big[V_iV_jB_{ij}\cos(\theta_i - \theta_j) \\ & - V_i^*V_j^*B_{ij}\cos(\theta_i^* - \theta_j^*)\Big] \\ & - \frac{1}{2}\sum_{i=1, i\neq n+1}^{n+m+1} [V_iV_{n+1}B_{i(n+1)}\cos(\theta_i - \theta_{n+1}) \\ & - V_i^*V_{n+1}^*B_{i(n+1)}\cos(\theta_i^* - \theta_{n+1}^*)], \quad k_{i1} > 0 \text{ is a scalar} \end{aligned}$$

According to the condition (2a) and (2b), the algebraic equations to be solved are

$$\begin{aligned} & -\frac{\omega_0}{M_i}\frac{\partial H_d}{\partial \delta_i} - \frac{\omega_0 D_i}{M_i^2}\frac{\partial H_d}{\partial \omega_i} + J_{i1}\frac{\partial H_d}{\partial E'_{qi}} \\ & = \frac{\omega_0}{M_i}(P_{mi} - P_{ei}) - \frac{D_i}{M_i}(\omega_i - \omega_0) \end{aligned} \tag{18}$$

$$\partial_z H_d(x,z) = A(x,z)\sigma(x,z) + B(x,z)G^{\mathrm{T}}\partial_x H_d(x,z) \tag{19}$$

where $x = [x_1, \ldots, x_n]^{\mathrm{T}}$, $x_i = [\delta_i, \omega_i, E'_{qi}]^{\mathrm{T}}$, $z = [z_1, \ldots, z_{n+m+1}]^{\mathrm{T}}$, $z_i = [\theta_i, V_i]^{\mathrm{T}}$,

$$G = \begin{bmatrix} G_1 & & \\ & \ddots & \\ & & G_n \end{bmatrix}, G_i = [0,0,1]^{\mathrm{T}}$$

Then we can get

$$J_{i1} = -\frac{\omega_0 k_{i1}}{M_i}V_i\sin(\delta_i - \theta_i)$$

$$A(x,z) = \mathbf{I}_{2(n+m)},$$

$$B(x,z) = \begin{bmatrix} b_{11}(x_1,z_1) & & \\ b_{12}(x_1,z_1) & & \\ & \ddots & \\ & \ddots & \\ & & b_{n1}(x_n,z_n) \\ & & b_{n2}(x_n,z_n) \\ & & \end{bmatrix} \tag{20}$$

where $\mathbf{I}_{2(n+m)}$ is the identity matrix, $b_{i1}(x_i,z_i) = k_{i1}V_i\sin(\delta_i - \theta_i)$, $b_{i2}(x_i,z_i) = k_{i1}\cos(\delta_i - \theta_i)$.

The Hamiltonian structure of system (10) can be expressed as

$$\begin{aligned} \dot{x}_i &= \begin{bmatrix} 0 & \frac{\omega_0}{M_i} & 0 \\ -\frac{\omega_0}{M_i} & -\frac{\omega_0 D_i}{M_i^2} & J_{i1} \\ 0 & -J_{i1} & -k_{ei} \end{bmatrix} \begin{bmatrix} \frac{\partial H_d}{\partial \delta_i} \\ \frac{\partial H_d}{\partial \omega_i} \\ \frac{\partial H_d}{\partial E'_{qi}} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u'_i \\ &= [J_i - R_i]\frac{\partial H_d}{\partial x_i} + G_i u'_i \end{aligned} \tag{21}$$

In the following, we check that the Hamiltonian function $H_d(x,z)$ satisfies the Lyapunov function properties.

The first property relates to the strict minimum at the given equilibrium (x^*, z^*).

Define the Jacobian

$$J_l = \begin{bmatrix} \frac{\partial g}{\partial \theta} & \frac{\partial g}{\partial V} \\ \frac{\partial \mathbf{h}}{\partial \theta} & \frac{\partial h}{\partial V} \end{bmatrix} \tag{22}$$

Definition 1. [20]: for the compact set C_k, det $J_l|_{(x,z)\in C_k} \neq 0$, and $J_l|_{(x,z)\in C_k}$ has k negative eigenvalues.

There are often several solutions to the equilibrium load flow equations of which one is the normal stable equilibrium point (SEP), angularly stable for the DAE model (15) and within the sheet C_0. Usually, the SEP in C_0 will be considered the 'normal' stable operating point of the power system. Other equilibrium points, which correspond to "low voltage" solutions, are usually considered to be infeasible even if they are angularly stable for the DAE model [20,21]. So it is reasonable to assume that the stable equilibrium $(x^*, z^*) \in C_0$.

Lemma 1. *The Hamiltonian function $H_d(x,z)$ defined by (17) admits a strict minimum at the stable equilibrium (x^*, z^*).*

The proof of this lemma is given in Appendix.

In order to ensure the validity of the Hamiltonian function $H_d(x,z)$ as a Lyapunov function, we also need to check that the derivative of $H_d(x,z)$ along trajectories is locally non-positive about the operating point.

Here, we determine an adequately large vector $\mathbf{m}_i = [m_{i1}, m_{i2}]^{\mathrm{T}}$ such that

$$|b_{i1}\dot{\theta}_i| \leqslant m_{i1}, \quad |b_{i2}\dot{V}_i| \leqslant m_{i2} \ \ \forall(\boldsymbol{x}, \boldsymbol{z}) \in \Omega \tag{23}$$

Following the Proposition 1, the excitation control law of generators can be expressed as

$$\begin{aligned} E_{fi} &= -(x_{di} - x'_{di})K_i(\delta_i, E'_{qi}, \theta_i, V_i) \\ &\quad + T'_{d0i}\left[-J_{i1}\frac{\partial H_d}{\partial \omega_i} - k_{ei}\frac{\partial H_d}{\partial E'_{qi}} + u'_i\right] \\ &= -(x_{di} - x'_{di})K_i(\delta_i, E'_{qi}, \theta_i, V_i) \\ &\quad + T'_{d0i}\left[k_{i1}V_i\sin(\delta_i - \theta_i)(\omega_i - \omega_0) - k_{ei}\frac{\partial H_d}{\partial E'_{qi}}\right. \\ &\quad \left. -\mathrm{sign}\left(\frac{\partial H_d}{\partial E'_{qi}}\right)(m_{i1} + m_{i2})\right] \end{aligned} \tag{24}$$

where $\frac{\partial H_d}{\partial E'_{qi}} = \gamma_i\left[E'_{qi} - E'^{*}_{qi} + k_{i1}[V_i\cos(\delta_i - \theta_i) - V_i^*\cos(\delta_i^* - \theta_i^*)]\right] + \frac{1}{x'_{di}k_{i1}}\left(E'_{qi} - E'^{*}_{qi}\right)$

Then the derivative of $H_d(\boldsymbol{x}, \boldsymbol{z})$ along trajectories is locally non-positive about the operating point.

$$\dot{H}_d \leqslant -\sum_{i=1}^{n}\frac{D_i}{\omega_0}(\omega_i - \omega_0)^2 - \sum_{i=1}^{n}k_{ei}\left(\frac{\partial H_d}{\partial E'_{qi}}\right)^2 \leqslant 0 \tag{25}$$

Because the system (15) is stable at the operating point, it can be see from the dynamic system theory that the system converges to the largest invariant set contained in

$$\begin{aligned} E &= \{\boldsymbol{x}, \boldsymbol{z} : \dot{H}_d(\boldsymbol{x}, \boldsymbol{z}) = 0\} = \{\boldsymbol{x}, \boldsymbol{z} : \omega_i = \omega_0, \\ E'_{qi} &= E'^{*}_{qi}, \boldsymbol{g} = 0, \boldsymbol{h} = 0, \quad i = 1, \ldots, n\} \end{aligned} \tag{26}$$

From $\omega_i \equiv \omega_0, E'_{qi} \equiv E'^{*}_{qi}$, we can conclude that $P_{mi} - P_{ei} = 0 \quad i = 1, \ldots, n$

Thus, the point in the largest invariant set satisfy

$$\begin{cases} \omega_i = \omega_0 \\ P_{mi} - P_{ei} = 0 \\ \boldsymbol{g} = 0 \\ \boldsymbol{h} = 0 \end{cases}, \quad i = 1, \ldots, n \tag{27}$$

which is exactly the condition the equilibrium satisfies. Hence there exists a suitably small neighborhood, Ω, of the operating point such that the largest invariant set in Ω only contains one point, i.e., the operating point. Because $H_d(\boldsymbol{x}, \boldsymbol{z})$ is positive definite about the equilibrium point, $H_d(\boldsymbol{x}, \boldsymbol{z})$ can be regarded as Lyapunov function. From the LaSalle's invariance principle, the closed-loop system (15) with the control law (24) ensure asymptotic stability of the desired equilibrium with the Hamiltonian function $H_d(\boldsymbol{x}, \boldsymbol{z})$.

Remark 1. The control law can be made more practicable for engineering applications.

According to the power system dynamic, we have

$$E'_{qi} \approx V_i + Q_{ei}x'_d/V_i, \delta_i = \int_0^t(\omega_i(\tau) - \omega_0)\,\mathrm{d}\tau \tag{28}$$

where Q_{ei} is the generator reactive power output.

According to the expression (28), the variables in the control law (24) are local measurable and just related to the same generator. Therefore, the proposed control law is decentralized and decoupled. It means the proposed method is not limited by the size of power system. Because the Hamiltonian function $H_d(\boldsymbol{x}, \boldsymbol{z})$ satisfies the Lyapunov function properties, the proposed method is still effective in case of a large power system according to Proposition 1.

While assuring the desired behavior, the control law (24) is discontinuous across the surface $\partial H_d/\partial E'_{qi} = 0$, which leads to control chattering. We can remedy this situation by smoothing out the control discontinuities in a boundary layer neighboring the surface $\partial H_d/\partial E'_{qi} = 0$. The sign function sign(·) in (24) is replaced by sat(·), where

$$\mathrm{sat}(s_i) = \begin{cases} 1 & \text{if } s_i > k_{i2} \\ s_i/k_{i2} & \text{if } -k_{i2} \leqslant s_i \leqslant k_{i2} \\ -1 & \text{if } s_i < -k_{i2} \end{cases} \tag{29}$$

4. Simulation

To illustrate the effect of the proposed method on transient stability, transient stability studies are made on a nine-bus system as shown in Fig. 1. The data of the generator and network are provided in [13].

In this example, Generator 1 is chosen as the reference machine. The static characteristic for each reactive power is represented as

$$Q_i^d(V_i) = Q_{i0}[0.2(V_i/V_{i0})^2 + 0.4(V_i/V_{i0}) + 0.4] \tag{30}$$

For the purpose of comparison, two different control schemes are investigated:

Scheme 1: Generator 2 and 3 are equipped with the same automatic voltage regulator (AVR) and different power system stabilizer (PSS). Fig. 2 shows the AVR model. The transfer function of conventional PSS with input signal $\Delta\omega$ is

$$G(s) = K_w\frac{sT_w}{1 + sT_w}\frac{(1 + sT_1)}{(1 + sT_2)}\frac{(1 + sT_3)}{(1 + sT_4)} \tag{31}$$

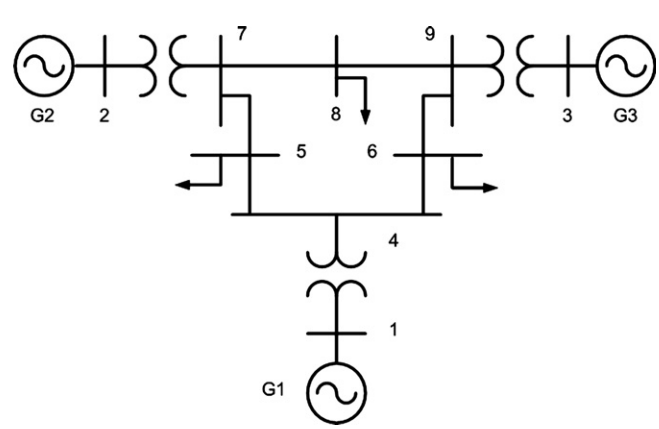

Fig. 1. A nine-bus power system.

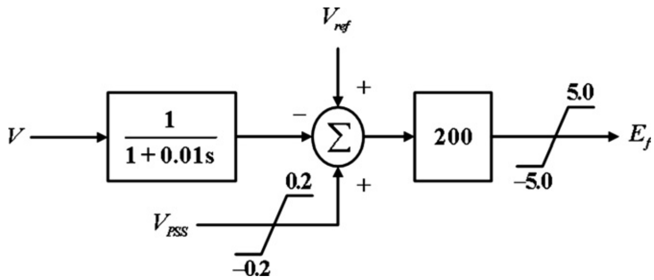

Fig. 2. Conventional automatic voltage regulator.

The data of PSS is shown in Table 1.

Scheme 2: The proposed IDA-PBC controller is implemented at Generator 2 and 3, and parameters are given in Table 2.

To simulate the system behaviour under large disturbance conditions, studies considering a wide variety of fault conditions have been carried out on the test system. All faults are assumed to be introduced at the same time $t = 0.2$ s and removed after a certain time (called clearing time t_{cl}). The following faults are considered:

Case 1: A fault occurs at bus 9. The pre-fault and post-fault systems are identical. $t_{cl} = 0.083$ s.

Case 2: Assume there are two lines between bus 5and bus 7. A fault occurs at bus 7 and is cleared by tripping one line between bus 5 and bus 7. $t_{cl} = 0.15$ s.

Case 3: The operating power level is increased by 30%. A fault occurs at bus 4. The pre-fault and post-fault systems are identical. $t_{cl} = 0.18$ s.

The simulation results are displayed for a period of 5 s. The control responses of two control schemes under the three fault conditions are shown in Figs. 3–9, respectively. It is seen that the generators oscillate initially when the disturbance occurs, and are then stabilized gradually within several seconds. The system is stable and damped quickly under the proposed IDA controller, whereas it continues for much longer under PSS control. From the simulation results it is observed that the proposed method can increase

Table 1
PSS data

	k_w	T_w	T_1	T_2	T_3	T_4
G_2	8.255	5	0.201	0.05	0.137	0.05
G_3	1.082	5	0.631	0.05	0.629	0.05

Table 2
The proposed controller data

	k_1	k_2	k_e	γ	m_1	m_2
G_2	6	0.05	20	0.1	9	1.2
G_3	4	0.05	25	0.1	6	0.8

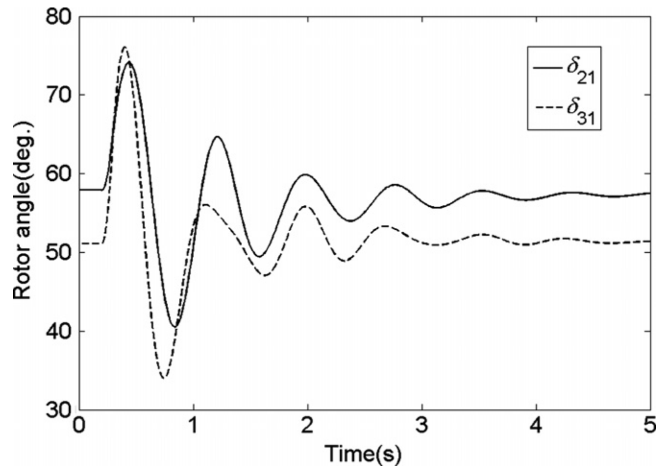

Fig. 3. Rotor angle response of the system with PSS (Case 1).

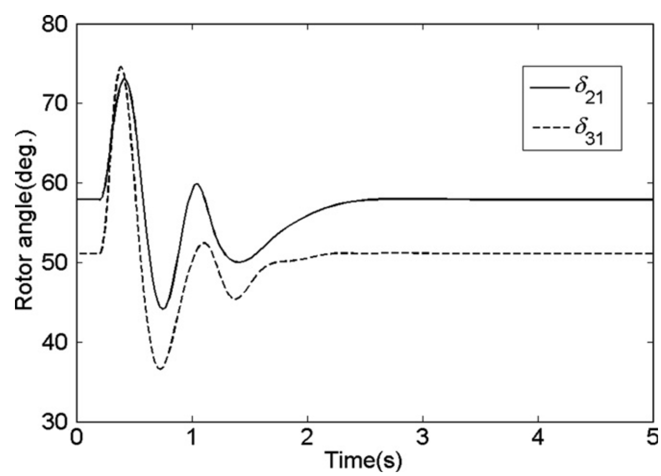

Fig. 4. Rotor angle response of the system with the proposed IDA-PBC (Case 1).

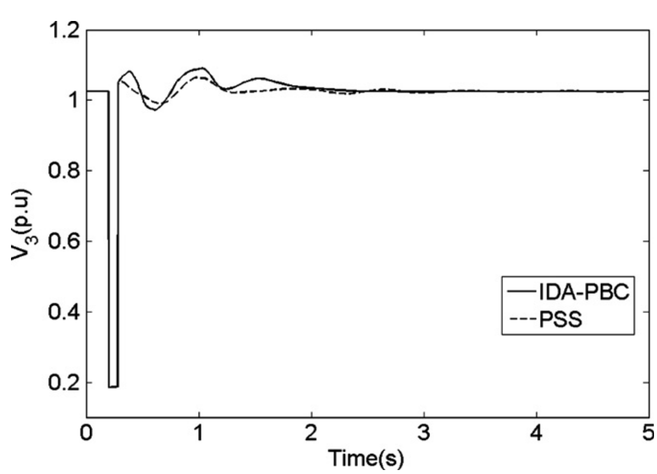

Fig. 5. Voltage response at bus 3 (Case 1).

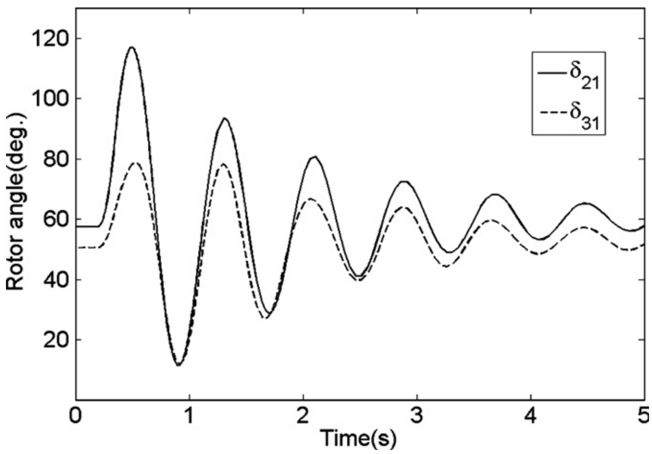

Fig. 6. Rotor angle response of the system with PSS (Case 2).

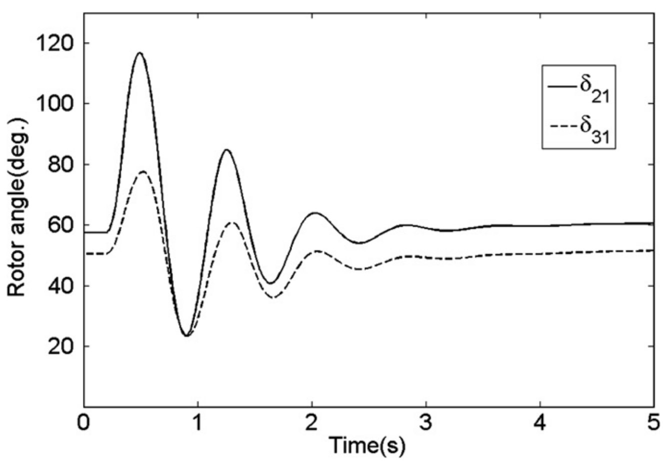

Fig. 7. Rotor angle response of the system with the proposed IDA-PBC (Case 2).

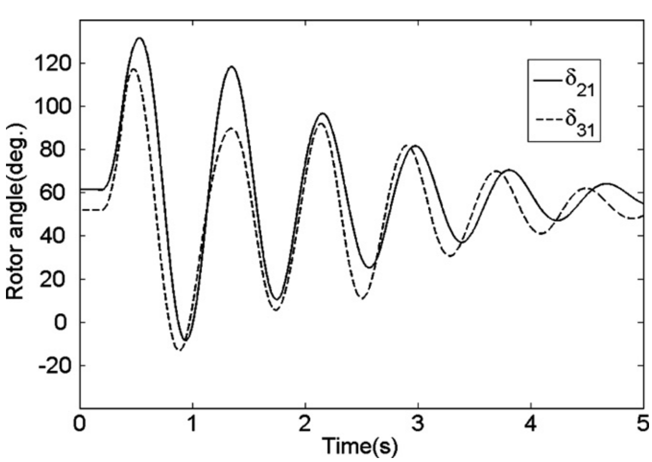

Fig. 8. Rotor angle response of the system with PSS (Case 3).

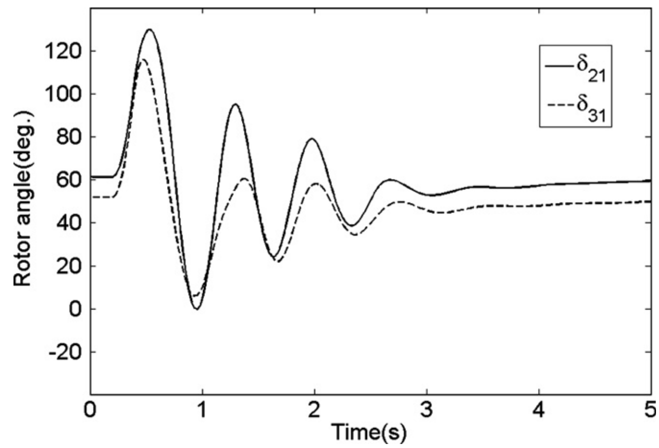

Fig. 9. Rotor angle response of the system with the proposed IDA-PBC (Case 3).

the system damping and improve the power angle stability of the power system with respect to different fault durations, fault locations. Further, we can also say that the proposed control law is effective at different operation levels compared with Scheme 1.

5. Conclusion

Recently developed IDA-PBC methodology is extended to study the stability and control of a class of DAE in this paper. To ensure asymptotic stability of the closed-loop system with a port-controlled Hamiltonian structure, a state feedback control law is proposed. And this method is applied to develop excitation controllers for multi-machine power systems based on structure preserving model. A key step in the procedure is the modification of the energy transfer between the electrical and the mechanical parts of the system. The physical features of the generator are used to construct Lyapunov function. Based on the dissipative Hamiltonian realization, a decentralized control strategy has been presented. The controller is proved to be effective in damping the oscillations and enhancing the system stability by the results of simulation research.

Appendix. Proof of Lemma 1

Since $H_1(\boldsymbol{x},\boldsymbol{z})$ is positive semi-definite and $H_1(\boldsymbol{x}^*,\boldsymbol{z}^*)=0$, in order to show the strong convexity of $H_d(\boldsymbol{x},\boldsymbol{z})$, we need to check the positive definiteness of $H_2(\boldsymbol{x},\boldsymbol{z})$.

At the equilibrium, we have

$$\left.\frac{\partial H_2}{\partial \boldsymbol{x}}\right|_{(\boldsymbol{x}^*,\boldsymbol{z}^*)}=0,\quad \left.\frac{\partial H_2}{\partial \boldsymbol{z}}\right|_{(\boldsymbol{x}^*,\boldsymbol{z}^*)}=0 \tag{A.1}$$

After a straightforward calculation and rearranging the rows and columns, the Hessian matrix of the function $H_2(\boldsymbol{x},\boldsymbol{z})$ is give by

$$Hess(H_2(\boldsymbol{x},\boldsymbol{z}))\Big|_{(\boldsymbol{x}^*,\boldsymbol{z}^*)} = \left[\begin{array}{c|c|c|cc} \frac{\partial \boldsymbol{P}}{\partial \boldsymbol{\delta}} & & & \frac{\partial \boldsymbol{P}}{\partial \boldsymbol{\theta}} & \frac{\partial \boldsymbol{P}}{\partial \boldsymbol{V}} \\ \hline & \boldsymbol{M}/\omega_0 & & & \\ \hline & & \boldsymbol{N} & & \\ \hline \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{\delta}} & & & & \\ \frac{\partial \boldsymbol{h}}{\partial \boldsymbol{\delta}} & & & \boldsymbol{J}_l & \end{array}\right]\Bigg|_{(\boldsymbol{x}^*,\boldsymbol{z}^*)}$$

where $\boldsymbol{M} = \begin{bmatrix} M_1 & & \\ & \ddots & \\ & & M_n \end{bmatrix}$, $\boldsymbol{N} = \begin{bmatrix} 1/(x'_{d1}k_{11}) & & \\ & \ddots & \\ & & 1/x'_{dn}k_{n1} \end{bmatrix}$,

$$\boldsymbol{P} = \boldsymbol{P}_e - \boldsymbol{P}_m \tag{A.2}$$

where $\mathbf{M} = \begin{bmatrix} M_1 & & \\ & \ddots & \\ & & M_n \end{bmatrix}$,

$$\boldsymbol{N} = \begin{bmatrix} 1/(x'_{d1}k_{11}) & & \\ & \ddots & \\ & & 1/x'_{dn}k_{n1} \end{bmatrix}$$

$$\boldsymbol{P} = \boldsymbol{P}_e - \boldsymbol{P}_m$$

Notice that Hessian matrix of $H_2(\boldsymbol{x},\boldsymbol{z})$ is positive definite iff

$$J = \left[\begin{array}{c|cc} \frac{\partial \boldsymbol{P}}{\partial \boldsymbol{\delta}} & \frac{\partial \boldsymbol{P}}{\partial \boldsymbol{\theta}} & \frac{\partial \boldsymbol{P}}{\partial \boldsymbol{V}} \\ \hline \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{\delta}} & & \\ \frac{\partial \boldsymbol{h}}{\partial \boldsymbol{\delta}} & \boldsymbol{J}_l & \end{array}\right]\Bigg|_{(\boldsymbol{x}^*,\boldsymbol{z}^*)} \tag{A.3}$$

is positive definite. According to the Schur complements, matrix J can only be positive definite iff $\boldsymbol{J}_l|_{(\boldsymbol{x}^*,\boldsymbol{z}^*)}$ and

$$\boldsymbol{F} = \frac{\partial \boldsymbol{P}}{\partial \boldsymbol{\delta}} - \begin{bmatrix} \frac{\partial \boldsymbol{P}}{\partial \boldsymbol{\theta}} & \frac{\partial \boldsymbol{P}}{\partial \boldsymbol{V}} \end{bmatrix} \boldsymbol{J}_l^{-1} \begin{bmatrix} \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{\delta}} \\ \frac{\partial \boldsymbol{h}}{\partial \boldsymbol{\delta}} \end{bmatrix}\Bigg|_{(\boldsymbol{x}^*,\boldsymbol{z}^*)} \tag{A.4}$$

are positive definite.

The matrix J_l is the Jacobian of the normal power flow Jacobian and $(\boldsymbol{x}^*,\boldsymbol{z}^*) \in C_0$. Therefore $\boldsymbol{J}_l|_{(\boldsymbol{x}^*,\boldsymbol{z}^*)}$ is positive definite [17,21].

Linearizing the structure preserving model with a constant quadrature axis voltage gives [20]

$$\begin{bmatrix} \Delta\dot{\boldsymbol{\delta}} \\ \Delta\dot{\boldsymbol{\omega}} \end{bmatrix} = \left[\begin{array}{c|c} 0 & \boldsymbol{I} \\ \hline -\boldsymbol{M}^{-1}\omega_0\boldsymbol{F} & -\boldsymbol{M}^{-1}\boldsymbol{D} \end{array}\right] \begin{bmatrix} \Delta\boldsymbol{\delta} \\ \Delta\boldsymbol{\omega} \end{bmatrix} = \boldsymbol{A}\begin{bmatrix} \Delta\boldsymbol{\delta} \\ \Delta\boldsymbol{\omega} \end{bmatrix}$$

At the operating point $(\boldsymbol{x}^*,\boldsymbol{z}^*)$, the linearized version of SPM is stable (angle-wise). It means that all eigenvalues of A have negative real parts. Because the number of positive eigenvalues of A is equal to the number of negative eigenvalues of F [20], F is positive definite.

Therefore, $H_2(x,z)$ has a strict local minimum at equilibrium point $(\boldsymbol{x}^*,\boldsymbol{z}^*)$, where $H_2(\boldsymbol{x}^*,\boldsymbol{z}^*) = 0$. It means $H_2(\boldsymbol{x},\boldsymbol{z})$ is positive definite. Hence, we can conclude that the Hamiltonian function $H_d(\boldsymbol{x},\boldsymbol{z})$ will be locally positive definite and reach the minimum at stable equilibrium point $(\boldsymbol{x}^*,\boldsymbol{z}^*)$.

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