Computers and Mathematics with Applications 64 (2012) 1343–1351

Contents lists available at SciVerse ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

On the exact expression for single-stage CSMA/CA's idle period distribution

Raymond J. Jayabal *, Chiew Tong Lau

School of Computer Engineering, Nanyang Technological University, Singapore

ARTICLE INFO

Keywords:
Wireless LAN
MAC
CSMA/CA
Idle period
Analytical model

ABSTRACT

The presence of CSMA/CA transmissions in a wired or wireless medium, as well as the number of active transmitters may be deducible via profiling energy level measurements in its band of operation. A distinct characteristic that may be useful in this application is its idle period distribution. In this paper, we propose an exact analytical model for this distribution for CSMA/CA with a single backoff stage, and verify its accuracy with simulation results alongside two approximation models.

1. Introduction

Carrier-Sense Multiple Access with Collision Avoidance (CSMA/CA) is a random access protocol that has been adopted in a number of local or personal area networks protocols such as Wi-Fi [1], Zigbee [2] and Homeplug. In CSMA/CA, nodes partake in a contention process involving the use of backoff counters and channel sensing prior to transmitting, as follows: At the end of a successful transmission process, a node will initialize its backoff counter to a uniformly random value within $[0, W_0 - 1]$, where W_0 is the contention window length. If it has a frame queued up for transmission, it will sense the channel for other nodes' transmissions, and decrement its counter for every unit of time known as the 'slot time' for which the channel is clear. When the node's counter reaches zero, it starts its transmission process. If the transmission is successful, the process is repeated for the next frame in its queue. If it is not, the process is repeated for the same frame with the counter value re-initialized to a uniformly random value in a range double the previous, i.e. $[0, 2W_0 - 1]$ if multiple, collision-based backoff stages with exponentially increasing windows are used, or in the same range if only a single stage is used.

Due to the mechanism of this contention process, CSMA/CA leaves a channel occupancy signature (Fig. 1) in its band of operation with an idle period distribution that can be mathematically expressed as a function of its contention window size and the number or participating nodes. Knowledge of this distribution may allow further analytical modeling of CSMA/CA in scenarios that are not possible with just the knowledge currently available in the literature, especially in situations involving co-channel interference or spatial reuse. For example, in [3], we explicitly use this distribution to develop the throughput model of CSMA/CA using a spatial channel-sharing scheme.

Apart from a purely analytical perspective, this distribution may also be useful in real-life applications whereby there is a need for a device to detect CSMA/CA transmissions and the number of actively participating nodes without fully implementing the demodulation scheme for the Physical layer protocol in use. For example, the device may employ a basic energy detector with frequency and band-pass filter set to the operating band under examination, collect the idle period statistics over time and perform best-of-fit tests with the curves generated by probability mass function of the idle period using different values for contention window size and number of nodes. Such applications may be common in devices that are has limited processing resources but are still required to perform its sensing functions across a wide range of protocols (e.g., spectrum sensing for cognitive radios, wireless sniffing, etc.).

* Corresponding author. Tel.: +65 91472486.
E-mail addresses: raym0002@ntu.edu.sg,jraymond@i2r.a-star.edu.sg, raymondjayaraj@yahoo.com.sg (R.J. Jayabal).

0898-1221/$ – see front matter
doi:10.1016/j.camwa.2012.03.078

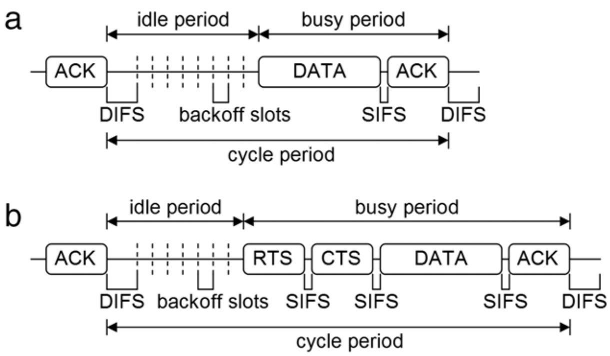

Fig. 1. Idle period in (a) CSMA/CA with ACK and (b) with RTS/CTS signaling.

In this paper, we propose an analytical model for the idle period distribution for CSMA/CA with a single backoff stage (i.e., with a fixed contention window size), and provide simulation results to demonstrate its accuracy.

2. Related work

Despite its maturity, there is still much to be uncovered regarding CSMA/CA from an analytical perspective. Year on year, new characteristics are discovered, such as the access delay and delay jitter [4,5], and collision probability in unsaturated loads [6]. We seek to add our discovery to this knowledge base. Our analysis is based on the Markov chain model of CSMA/CA that was initially proposed in [7] and subsequently improved upon in terms of correctness in [8–11]. A component of our model (the frozen counter distribution) was presented in [12]. The full idle period model incorporating a simpler and more intuitive model for that component appears in [13] in shortened form. In this extended paper we provide detailed elaboration of the derivation steps, provide more verification with simulation data (including Chi-Square test results), and further compare its accuracy against two approximation models for the idle period — one by Bowden et al. [14], and the other which can be intuitively worked out from the Markov chain model of CSMA/CA [7,8,11].

3. System assumptions

As mentioned, our analysis is for the case when the contention window is fixed. As a result, effects such as capture or frame outage due to fading will not affect its validity. However, any node within the network must be able to sense the transmissions of other active nodes in the network, i.e., there are no hidden nodes. Otherwise, the assumption that the backoff counter is decremented when the channel is clear (as per the protocol definition) will not be correct. In addition, active nodes are assumed to be having at least one frame in its transmission queue during the analysis period. Vice-versa, inactive nodes are assumed completely silent throughout the analysis period. Transmissions may be point-to-multipoint as in an infrastructure network, or peer-to-peer as in an ad hoc network.

During the busy period, there may be gaps (e.g., SIFS in Fig. 1). Such gaps are assumed to be smaller than the smallest possible idle period (e.g. DIFS), and are not considered in our analysis. For the sake of generality and easier mathematical representation, we denote the idle period, I, as a discrete random variable representing the number of backoff slots at the start of the CSMA/CA cycle (Fig. 1). The actual observable idle period, I_{actual}, can be obtained by multiplying I with the backoff slot duration, and adding in any fixed period that may precede it, e.g. $I_{\text{actual}} = I \times \text{aSlotTime} + \text{DIFS}$ as per the IEEE 802.11 standard. Based on this convention, for CSMA/CA with fixed contention window, the value of I will be bounded in $[0, W_0 - 1]$ since the backoff counter can only take values in this range.

4. Proposed model

Denote B_n as the random variable representing the new value that a node sets its backoff counter to after a transmission, and B_f the value at which it is frozen when it senses the channel as busy. At the end of each cycle, there will be between one to N nodes transmitting in the busy period for that cycle (resulting in a collision if more than one node is involved). Let $\Gamma_{[1,N]}$ be the variable for this value. Assuming $\Gamma_{[1,N]}$ and B_f are not interdependent, it can be worked out that I has the distribution,

$$\Pr(I = i) = \frac{\sum_{\tau=1}^{N}[\Pr(\Gamma_{[1,N]} = \tau) \cdot \Pr(B_n \geq i)^{\tau} \cdot \Pr(B_f \geq i)^{N-\tau} \cdot (1 - \Pr(B_n > i \mid B_n \geq i)^{\tau} \cdot \Pr(\mathrm{B}_f > i \mid B_f \geq i)^{N-\tau})]}{\sum_{\tau=1}^{N} \Pr(\Gamma_{[1,N]} = \tau)}$$

$$i \in [0, W_0 - 1]. \tag{1}$$

The above equation simply states that, for the idle period to be i slots, the backoff counter for all nodes at the end of the previous cycle must be equal to or greater than i, with at least one node having its backoff counter value equal i. It is made up of three distributions: $\Gamma_{[1,N]}$, B_n and B_f. The distribution of $\Gamma_{[1,N]}$ can be obtained through solving the Markov chain model of the number of transmitters $\Gamma \equiv \Gamma_{[0,N]}$ at the turn of each slot. Γ's state transition probabilities can be worked out from [11] to be:

$$\Pr(\Gamma_{k+1} = \tau_1 \mid \Gamma_k = \tau_0) = \begin{cases} \binom{N}{\tau_1}\left(\frac{2}{W_0}\right)^{\tau_1}\left(\frac{W_0-2}{W_0}\right)^{N-\tau_1} & \tau_0 = 0,\ \tau_1 \in [0, N] \\ \binom{\tau_0}{\tau_1}\left(\frac{1}{W_0}\right)^{\tau_1}\left(\frac{W_0-1}{W_0}\right)^{\tau_0-\tau_1} & \tau_0 \in [1, N],\ \tau_1 \in [0, \tau_0] \\ 0 & \tau_0 \in [1, N],\ \tau_1 \in [\tau_0 + 1, N]. \end{cases} \tag{2}$$

Its stationary distribution, $\underline{\pi}_\Gamma$ is solvable with,

$$\underline{\pi}_\Gamma = \lim_{n\to\infty} [1\ 0\ \cdots] \cdot \underline{P}^n \tag{3}$$

where the i-th element of row vector $\underline{\pi}_\Gamma$ equals $\Pr(\Gamma = i)$, and the (i, j)-th element of matrix $\underline{P}$ equals $\Pr(\Gamma_{k+1} = j \mid \Gamma_k = i)$. $\Gamma_{[1,N]}$ is simply equal to Γ minus the case when $\Gamma = 0$, i.e.,

$$\begin{aligned} \Pr(\Gamma_{[1,N]} = \tau) &= \Pr(\Gamma = \tau \mid \Gamma > 0) \\ &= \frac{\Pr(\Gamma = \tau)}{1 - \Pr(\Gamma = 0)} \quad \tau \in [1, N]. \end{aligned} \tag{4}$$

Meanwhile, the distribution for B_n by definition of the protocol is,

$$\Pr(B_n = b) = \frac{1}{W_0} \quad b \in [0, W_0 - 1]. \tag{5}$$

This leaves us with just one more distribution to solve: B_f, the frozen counter distribution.

4.1. Frozen counter distribution

Our derivation of the frozen counter distribution is based on the observation in [8] and [14] that at the start of any backoff slot, the distribution of the backoff counter value for a node takes on the left-handed triangular distribution in $[0, W_0 - 1]$. We can use this observation to derive B_f: Since a node freezes its counter on detecting transmissions from other nodes, it is reasonable to assume that the distribution for B_f is also a left-handed triangular distribution, but in the range $[1, W_0 - 1]$ since at the value of zero, the node itself will transmit:

$$\Pr(B_f = b) = \Pr(B_f = W_0 - 1) + m \cdot (W_0 - 1 - b) \quad b \in [1, W_0 - 1] \tag{6}$$

$$\sum_{b=1}^{W_0-1} \Pr(B_f = b) = 1. \tag{7}$$

Eliminating gradient m from (1) using (7) we obtain:

$$\Pr(B_f = b) = \begin{cases} \frac{2(W_0 - 1 - b)}{(W_0 - 1)(W_0 - 2)} + \frac{2b - W_0}{W_0 - 2} \cdot \Pr(B_f = W_0 - 1) & W_0 > 2 \\ 1 & W_0 = 2 \end{cases} \quad b \in [1, W_0 - 1]. \tag{8}$$

The second clause in the above equation has the value '1' because with $W_0 = 2$, B_f can only assume one value, i.e. '1'. Note that (2) has an unknown parameter in $\Pr(B_f = W_0 - 1)$. For large windows, this may be approximated to zero. But this will not hold for smaller window sizes. Having an accurate value for this point in B_f's distribution is the key to having an accurate formula for the idle period distribution.

4.1.1. Probability $\Pr(B_f = W_0 - 1)$

For B_f to take the value of $(W_0 - 1)$, two events must occur consecutively following a collision: Firstly, a collided node must select $(W_0 - 1)$ as its new counter value at the end of a cycle in which it had transmitted. Secondly, in the following cycle, the other node (or nodes) involved in the collision transmits immediately, leaving no idle slots for the first node to decrement its counter before freezing it. This results in B_f assuming the same value as B_n. If the third cycle has no idle slots as well, then the first node will freeze its counter at $W_0 - 1$ again, and repeatedly do so as long as subsequent cycles have no idle slots.

This situation of a node having its counter frozen at its new value is not exclusive to the case when the new counter value is $W_0 - 1$: as long as the new value is within $[1, W_0 - 1]$, the above scenario can occur. In order to solve for the specific case when the new selection equals $W_0 - 1$, we must first consider this general case. By set convention, this situation can be represented as $\{B_f = b_n \mid B_n = b_n\}$. The flipside of it, i.e., the situation when the counter is not frozen at its new value, is denoted as $\{B_f \neq b \mid B_n = b\}$, or more precisely, $\{B_f < b \mid B_n = b\}$ since $B_f < B_n$. This situation is only possible for nodes that go through a cycle without transmitting in it. Denote α and β as the average times these situations occur respectively, over a renewal interval comprising one cycle with an idle period followed by K 'busy-period-only' cycles. The number of transmitters in these K 'busy-period-only' cycles follow the sequence $\{\Gamma_0, \Gamma_1, \Gamma_2, \ldots, \Gamma_k, \Gamma_{k+1}, \ldots, \Gamma_{K-1}\}$ where Γ_k represents the number of transmitting nodes in the k-th cycle in the interval, with K in $[1, \infty]$. If α and β is found, then we would have solved $\Pr(B_f = W_0 - 1)$ since,

$$\Pr(B_f = W_0 - 1) = \Pr(B_f = W_0 - 1 \mid B_n > 0) \cdot \frac{\alpha}{\alpha + \beta} = \frac{1}{W_0 - 1} \cdot \frac{\alpha}{\alpha + \beta} \tag{9}$$

4.1.2. Average times for $\{B_f = b \mid B_n = b\}$

Denote $\alpha_{\tau 0}$ as the number of times $\{B_f = b \mid B_n = b\}$ occur in the interval which starts with $\Gamma_0 = \tau_0$. Obviously, with $\tau_0 = 1$, $\{B_f = b \mid B_n = b\}$ cannot happen in the subsequent cycle since at least 2 nodes are required to make it happen (i.e., one node to select a non-zero value and the other to select zero for their backoff counters). Hence, $\alpha_1 = 0$.

With $\tau_0 = 2$, $\{B_f = b \mid B_n = b\}$ can only occur if there is a 'drop' from 2 to 1 in the sequence for Γ_k. For example, sequences {2, 1}, {2, 1, 1}, {2, 2, 2, 1, 1} all result in one node freezing its backoff counter at its initial value in each cycle after the 'drop'. Over all possible sequences matching the general pattern {2..., 1...}, the average times $\{B_f = b \mid B_n = b\}$ occur is therefore:

$$\alpha_2 = \sum_{i=0}^{\infty} [\Pr(\Gamma_k = 2 \mid \Gamma_{k-1} = 2)]^i \cdot \Pr(\Gamma_k = 1 \mid \Gamma_{k-1} = 2) \cdot \sum_{i=0}^{\infty} [\Pr(\Gamma_k = 1 \mid \Gamma_{k-1} = 1)]^i = \frac{\Pr(\Gamma_k = 1 \mid \Gamma_{k-1} = 2)}{[1 - \Pr(\Gamma_k = 2 \mid \Gamma_{k-1} = 2)] \cdot [1 - \Pr(\Gamma_k = 1 \mid \Gamma_{k-1} = 1)]}. \tag{10}$$

The numerator in the above equation is the probability that Γ_k transitions from 2 to 1 at some point along the sequence, while the denominator terms are the probabilities that Γ_k is stuck at 1 and 2 for all possible number of repetitions.

With $\tau_0 = 3$, three sets of 'busy-period-only' cycle sequences for Γ_k are possible which contains situation: {3..., 2...}, {3..., 1...} and {3..., 2..., 1...}. The first set will have one node repeatedly encountering this situation, while the last two sets will have 2 nodes doing so. Since the last set includes the pattern {2...,1...}, α_2 can be incorporated into the formulation for α_3 as follows:

$$\alpha_3 = \frac{\Pr(\Gamma_k = 2 \mid \Gamma_{k-1} = 3)}{[1 - \Pr(\Gamma_k = 3 \mid \Gamma_{k-1} = 3)] \cdot [1 - \Pr(\Gamma_k = 2 \mid \Gamma_{k-1} = 2)]} + 2.\frac{\Pr(\Gamma_k = 1 \mid \Gamma_{k-1} = 3)}{[1 - \Pr(\Gamma_k = 3 \mid \Gamma_{k-1} = 3)] \cdot [1 - \Pr(\Gamma_k = 1 \mid \Gamma_{k-1} = 1)]} + 2.\frac{\Pr(\Gamma_k = 2 \mid \Gamma_{k-1} = 3) \cdot \alpha_2}{1 - \Pr(\Gamma_k = 3 \mid \Gamma_{k-1} = 3)}. \tag{11}$$

With increasing values of τ_0, it can be shown that the recursive function:

$$\alpha_{rec}(\tau_{iter}, \tau_0) = \frac{\left\{ \sum_{i=1}^{\tau_{iter}-1} \left[\frac{(\tau_0 - i) \cdot \Pr(\Gamma_k = i \mid \Gamma_{k-1} = \tau_{iter})}{1 - \Pr(\Gamma_k = i \mid \Gamma_{k-1} = i)} \right] + \sum_{i=2}^{\tau_{iter}-1} [\Pr(\Gamma_k = i \mid \Gamma_{k-1} = \tau_{iter}) \cdot \alpha_{rec}(i, \tau_0)] \right\}}{1 - \Pr(\Gamma_k = \tau_{iter} \mid \Gamma_{k-1} = \tau_{iter})} \tag{12}$$

yields $\alpha_{\tau 0}$ if we start the recursion with $\tau_{iter} = \tau_0$. α which is the parameter we are originally interested in, is the sum of $\alpha_{\tau 0}$ over all possible values of τ_0, weighted by the probability that the interval starts τ_0 with $\Gamma_0 = \tau_0$:

$$\alpha = \sum_{\tau_0=2}^{N} [\Pr(\Gamma_k = \tau_0 \mid \Gamma_{k-1} = 0) \cdot \alpha_{\tau_0}]. \tag{13}$$

4.1.3. Average times for $\{B_f < b \mid B_n = b\}$

β can similarly be worked out following the recursive approach used to solve for α. Denote $\beta_{\tau 0}$ as the number of times $\{B_f < b \mid B_n = b\}$ is experienced by a node in the interval. Observing all possible sequences with $\tau_0 = 1$, i.e., sequences which match the pattern {1, 1, ... 1}, it can be worked out that:

$$\beta_1 = \sum_{i=0}^{\infty} [\Pr(\Gamma_k = 1 \mid \Gamma_{k-1} = 1)]^i = \frac{1}{1 - \Pr(\Gamma_k = 1 \mid \Gamma_{k-1} = 1)}. \tag{14}$$

With $\tau_0 = 2$, two general patterns are possible: {…2…} and {…2, 1….}. For this case, it can be worked out that:

$$\beta_2 = \sum_{i=0}^{\infty}[\Pr(\Gamma_k = 2 \mid \Gamma_{k-1} = 2)]^i + \sum_{i=0}^{\infty}[\Pr(\Gamma_k = 2 \mid \Gamma_{k-1} = 2)]^i \cdot \Pr(\Gamma_k = 1 \mid \Gamma_{k-1} = 2) \cdot \beta_1$$
$$= \frac{1 + \Pr(\Gamma_k = 1 \mid \Gamma_{k-1} = 2) \cdot \beta_1}{1 - \Pr(\Gamma_k = 2 \mid \Gamma_{k-1} = 2)}. \quad (15)$$

Continuing along this route of increasing values for τ_0, it can be worked out that $\beta_{\tau 0}$ can be solved recursively with,

$$\beta_{\tau_0} = \frac{1 + \sum_{i=1}^{\tau_0 - 1}[\Pr(\Gamma_k = i \mid \Gamma_{k-1} = \tau_0).\beta_i]}{1 - \Pr(\Gamma_k = \tau_0 \mid \Gamma_{k-1} = \tau_0)}. \quad (16)$$

β which is the parameter we are primarily interested in, is the sum of $\beta_{\tau 0}$ over all values of τ_0 for which $\{B_f < b \mid B_n = b\}$ is possible (i.e.,τ_0 in $[1, N-1]$), weighted by the probability that the interval starts with $\Gamma_0 = \tau_0$ and the number of nodes not transmitting in the interval:

$$\beta = \begin{cases} \sum_{\tau_0=1}^{N}[\Pr(\Gamma_k = \tau_0 \mid \Gamma_{k-1} = 0) \cdot (N - \tau_0) \cdot \beta_{\tau_0}] & W_0 > 2 \\ 0 & W_0 = 2. \end{cases} \quad (17)$$

The second clause in the above equation equals '0', since $\{B_f < b \mid B_n = b\}$ is not possible when $W_0 = 2$. With this equation, the frozen counter distribution, B_f, is solved: α and β obtained via (13) and (17) can be inserted into (9) to obtain Pr $(B_f = W_0 - 1)$, which in turn can be substituted into (2) to obtain the frozen counter distribution, Pr $(B_f = b)$:

$$\Pr(B_f = b) = \begin{cases} \frac{2(W_0 - 1 - b)}{(W_0 - 1)(W_0 - 2)} + \frac{2b - W_0}{(W_0 - 1)(W_0 - 2)} \cdot \frac{\alpha}{\alpha + \beta} & W_0 > 2 \\ 1 & W_0 = 2 \end{cases}$$
$$= \begin{cases} \frac{\frac{1}{W_0 - 1} \cdot \alpha + \frac{2(W_0 - 1 - b)}{(W_0 - 1)(W_0 - 2)} \cdot \beta}{\alpha + \beta} & W_0 > 2 \\ 1 & W_0 = 2 \end{cases} \quad b \in [1, W_0 - 1]. \quad (18)$$

This, and the new counter distribution Pr $(B_n = b)$ given in (5), can be substituted into (1) to get the idle period distribution.

5. Alternative model 1: Bowden et al.'s approximation

Although the analysis and verification in [14] was for the average idle period, Bowden et al. did provide an approximation for the full distribution as an intermediary step. Their analysis is based on approximating the discrete backoff counter distribution as a continuous one, and further making two simplifying assumptions: Firstly, it is assumed the new backoff counter value is randomly polled from $[1, W_0]$. Based on this assumption, the continuous cumulative density function approximating the new and running backoff counter distribution was given respectively as:

$$B_{n_{cum}}(b) = 1 - \frac{W_0 - b}{W_0} \quad 0 \le b \le W_0 \quad (19)$$

$$B_{f_{cum}}(b) = 1 - \left(\frac{W_0 - b}{W_0}\right)^2 \quad 0 \le b \le W_0. \quad (20)$$

Secondly, it was assumed that in any CSMA/CA cycle, the busy period involves only one transmitter. As such, the probability of the idle period being less than i was given as the product of $(N - 1)$ stations having less than i for their frozen counter values and one station having less than i for its new counter value:

$$I_{cum}(i) = 1 - (1 - B_{f_{cum}}(i))^{N-1}(1 - B_{n_{cum}}(i))$$
$$= 1 - \left(\frac{W_0 - i}{W_0}\right)^{2N-1} \quad 0 \le i \le W_0. \quad (21)$$

The discrete idle period distribution was then given as:

$$\Pr(I = i) = I_{cum}(i) - I_{cum}(i - 1) \quad i \in [1, W_0]. \quad (22)$$

To reconcile this formula to the fact that the new backoff counter falls within $[0, W_0 - 1]$, the frozen counter within $[1, W_0 - 1]$, and the idle period within $[0, W_0 - 1]$ rather than all falling within $[1, W_0]$. we modify the analysis by shifting the continuous distributions B_{ncum} and B_{fcum} accordingly:

$$B'_{n_{cum}}(b) = 1 - \frac{W_0 - 1 - b}{W_0} \quad -1 \le b \le W_0 - 1 \tag{23}$$

$$B'_{f_{cum}}(b) = 1 - \left(\frac{W_0 - 1 - b}{W_0 - 1}\right)^2 \quad 0 \le b \le W_0 - 1. \tag{24}$$

This modification leads to the following corrected pair of equations for the idle period distribution:

$$I'_{cum}(i) = \begin{cases} 1 - \dfrac{(W_0 - 1 - i)^{2N-1}}{W_0 (W_0 - 1)^{2N-2}} & 0 \le i \le W_0 - 1 \\ 1 - \dfrac{W_0 - 1 - i}{W_0} & -1 \le i \le 0 \end{cases} \tag{25}$$

$$\Pr(I = i) = I'_{cum}(i) - I'_{cum}(i - 1) \quad i \in [0, W_0 - 1]. \tag{26}$$

6. Alternative model 2: Markov chain approximation

Intuitively, the idle period distribution can also be approximated via (2) and (3). Here, we assume that each variate i of the idle distribution has the occurrence probability of a sequence of consecutive idle states of length $(i - 1)$ following a busy period. E.g., to evaluate the probability for Pr $(I = 3)$, we work out the chain probability for the sequence $\Gamma = \{\tau_k, 0, 0, 0, \tau_{k+4}\}$, where τ_k and τ_{k+4} are non-zeroes. Based on this logic, the idle period distribution following a busy period involving τ transmitters can thus be expressed as:

$$\Pr(I = i \mid \Gamma = \tau) = \begin{cases} \Pr(\Gamma_k > 0 \mid \Gamma_{k-1} = \tau) & i = 0 \\ \dfrac{[\Pr(\Gamma_k = 0 \mid \Gamma_{k-1} = \tau) \cdot \Pr(\Gamma_k = 0 \mid \Gamma_{k-1} = 0)^{(i-1)} \cdot \Pr(\Gamma_k > 0 \mid \Gamma_{k-1} = 0)]}{\Pr(\Gamma_k > 0 \mid \Gamma_{k-1} = 0) \cdot \sum_{\lambda=1}^{W_0-1} \Pr(\Gamma_k = 0 \mid \Gamma_{k-1} = 0)^{(\lambda-1)}} & i \in [1, W_0 - 1]. \end{cases} \tag{27}$$

The denominator term in the second clause for the above equation is just for normalizing the cumulative probability for the above equation to 1. It accounts for the fact that in the Markov chain model, the idle period can be infinitely long, whereas in the actual case its upper bound is $(W_0 - 1)$. Over busy periods involving all number of transmitters, the idle period distribution is therefore:

$$\Pr(I = i) = \frac{\sum_{\tau=1}^{N} \Pr(\Gamma = \tau) \cdot \Pr(I = i \mid \Gamma = \tau)}{1 - \Pr(\Gamma = 0)}. \tag{28}$$

7. Verification with simulation data

The ns-2 IEEE 802.11b simulator [15] was used to obtain the simulation results with which to assess the accuracy of our proposed model compared to Bowden et al.'s and the Markov chain approximations. Using default transceiver settings, N nodes were placed arbitrarily within a 50 m cell radius (well within sensing distance) to avoid the occurrence of hidden nodes, and fed with saturated peer-to-peer flows of 512-byte frames. A modification was made to the mac-802_11.cc source code to fix the contention window, and to record the idle periods into frequency bins in the range of $[0, W_0 - 1]$.

For each combination of W_0 in {4, 8, 16, 32, 64} and N in {2, 4, 6, 8, 10} we ran the simulation 30 times with each run starting with a different seed value for the random value generator, recording 10,000 samples of the idle period at each run. Using these 30 sets of data (one from each run), we then evaluate the accuracies of the three analytical models for each $W_0$and N setting. We present a few representative snapshots of the results in the following figures and tables.

In Fig. 2 the idle period distributions for W_0 in {4, 8, 16, 32} and N in {2, 10} based on the average of the 30 simulation results, and values computed using the three different models are shown. In general, the curve starts with a non-zero value at $(I = 0)$, abruptly peaks at $(I = 1)$ and then tapers off slowly towards the right end of the distribution. As N increases, the peak becomes narrower, but remains located at $(I = 1)$. The reason why the curve is shaped as such is largely due to the left-handed triangular shape of the frozen counter distribution which peaks at $(B_f = 1)$. Although this latter distribution tapers off linearly to the right, the probability distribution for at least one node having a certain frozen counter value in a

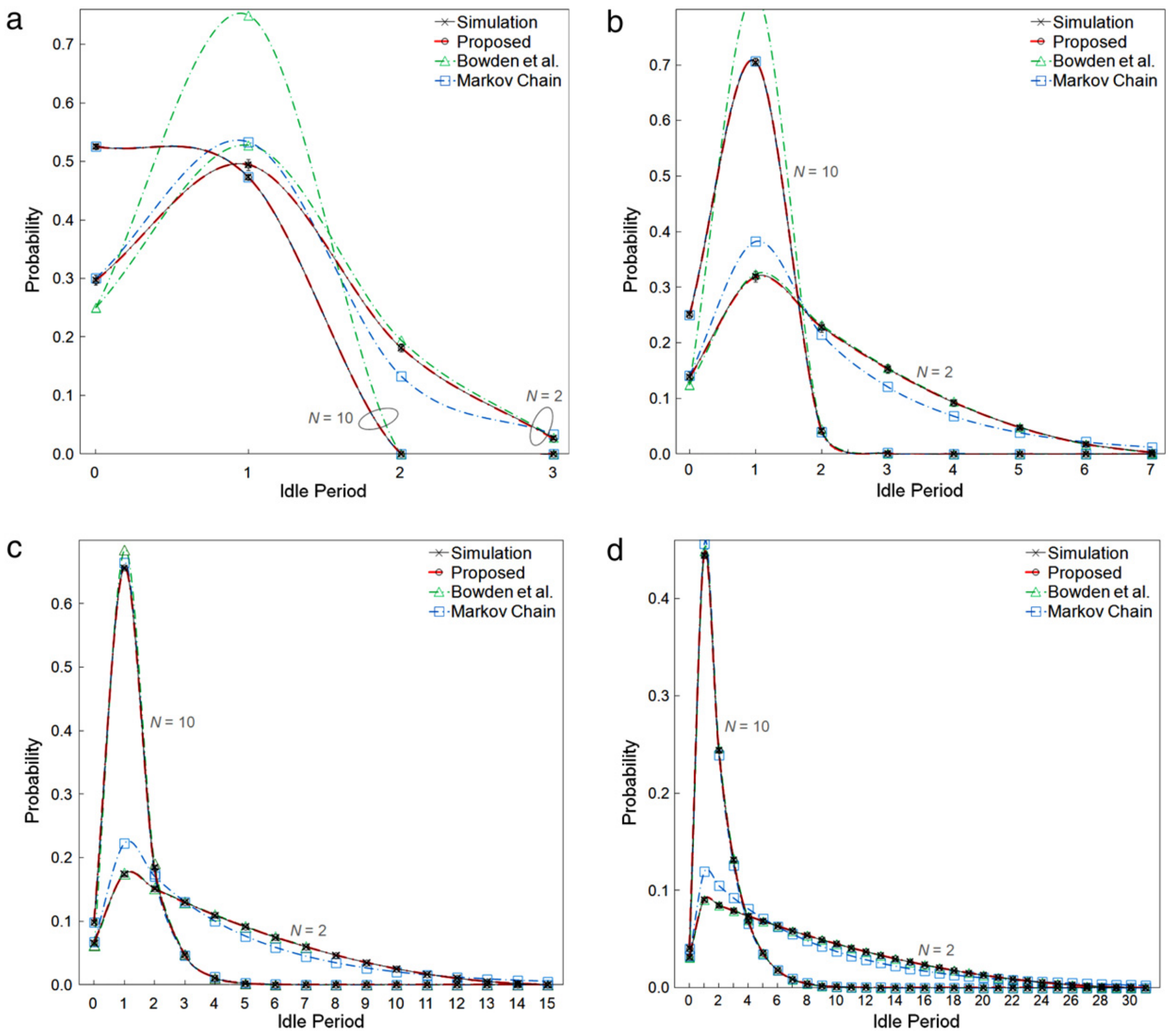

Fig. 2. Analytical vs. simulation values for Pr $(I = i)$ for (a) $W_0 = 4$, (b) $W_0 = 8$, (c) $W_0 = 16$ and (d) $W_0 = 32$.

group of N nodes would also end up with a peak at $(B_f = 1)$, but with a concaved taper that slopes off at a sharper angle as N increases.

Note that in all four charts, the proposed model yields curves that are practically coincident on the simulation curves. The Markov chain and Bowden et al.'s approximation models yield small degrees of deviation from the simulation curve when both values for N and W_0 are very small (e.g., $W_0 = 4$ and $N = 2$). However, as N gets larger while keeping to small values for W_0, the deviation for Bowden et al.'s method is aggravated immensely, whereas the accuracy for the Markov chain model improves to the point of nearly matching the simulation curve. This trend continues in their curves for $W_0 = 8$. However, with increasing W_0s, the accuracy of Bowden et al.'s model improves to the point of matching the simulation curves when $W_0 = 32$, whereas the Markov chain model continue to exhibit noticeable deviations.

Table 1 quantitatively confirms our observations regarding the relative accuracies of the models for small W_0s based on the charts. Besides points in the distribution, we also include the expectation, variance, and the Pearson's Chi-Square (χ^2) average test measure and passing rates (at p-value >0.05) for comparison. Here, it can be seen that for both $N = 2$ and $N = 10$, the proposed model yields values that fall within the 95% confidence interval (± 1.96 standard deviation) of the 30 sets of data, and passes all or nearly all Chi-Square tests on the 30 sets of data. For the Markov Chain model, this level of accuracy only seen when $N = 10$. Meanwhile, Bowden et al.'s model shows significant deviations from the 95% confidence interval of the simulation data, especially for $N = 10$.

As indicated by the graph for $W_0 = 32$, with the exception of the Markov Chain model, results for the other two models fall or converge on the simulation results for larger W_0s. This is demonstrated numerically in Table 2 which shows the values for both models falling within the 95% confidence band of the simulation results and having high passing rates for the Chi-Square test. Meanwhile, the numbers for the Markov Chain method suggest that this method may not be suitable for large W_0s.

Table 1
Analysis vs. simulation: idle period distribution, expectation, variance, chi-square test measure and pass rates for $W_0 = 4$ and $N = \{2, 10\}$.

N	Method	$\Pr(I) =$				$E[I]$	Var[I]	Avg. χ^2	% Pass
		(0)	(1)	(2)	(3)				
2	Simulation (95% CI)	0.295 0.299	0.492 0.496	0.181 0.184	0.026 0.027	0.935 0.942	0.578 0.585	–	–
	Proposed	**0.297**	**0.495**	**0.182**	**0.026**	**0.937**	**0.579**	**2.80**	**96.7**
	Bowden	**0.250**	**0.528**	**0.194**	**0.028**	**1.000**	**0.556**	**121.77**	**0.0**
	Markov	**0.300**	**0.533**	**0.133**	**0.033**	**0.900**	**0.557**	**219.07**	**0.0**
10	Simulation (95% CI)	0.524 0.528	0.472 0.475	0.000 0.001	0.000 0.000	0.473 0.476	0.250 0.251	–	–
	Proposed	**0.526**	**0.473**	**0.000**	**0.000**	**0.474**	**0.250**	**0.57**	**100.0**
	Bowden	**0.250**	**0.750**	**0.000**	**0.000**	**0.750**	**0.188**	**4072.61**	**0.0**
	Markov	**0.526**	**0.473**	**0.000**	**0.000**	**0.474**	**0.250**	**0.57**	**100.0**

Table 2
Analysis vs. simulation: idle period expectation, variance, chi-square test measure and pass rates for $W_0 = 64$ and $N = \{2, 10\}$.

N	Method	E[I]	Var[I]	Avg. χ^2	% Pass
2	Simulation (95% CI)	15.945 16.049	149.170 151.805	–	–
	Proposed	**15.996**	**150.560**	**56.82**	**100.0**
	Bowden	**16.000**	**150.534**	**56.76**	**100.0**
	Markov	**14.835**	**173.358**	**540.93**	**0.0**
10	Simulation (95% CI)	3.599 3.621	8.866 9.081	–	–
	Proposed	**3.610**	**8.987**	**18.90**	**100.0**
	Bowden	**3.618**	**8.971**	**21.87**	**93.3**
	Markov	**3.610**	**9.899**	**41.12**	**23.3**

Table 3
Chi-square test results on the idle period distribution models for all combinations of $W_0 = \{4, 8, 16, 32, 64\}$ and $N = \{2, 4, 6, 8, 10\}$.

Model	Avg. χ^2	% Pass
Proposed	**15.6**	**93.9**
Bowden	**562.0**	**33.7**
Markov	**130.0**	**18.0**

To conclude our analysis of the results, in Table 3 we rank the methods based on their per-test average χ^2 scores and passing rates (at p-value >0.05) for Chi-Square tests done for 30 simulation runs for each combination of $W_0 = \{4, 8, 16, 32, 64\}$ and $N = \{2, 4, 6, 8, 10\}$ (for a total of 750 tests).

8. Conclusion

We have presented an analytical model of the idle period distribution in CSMA/CA with a single backoff stage. Against simulation results, this model appears to be very accurate if not exact, judging from the results of the Pearson's Chi-Square goodness-of-fit tests under the range of settings for contention window size and number of nodes that we used. This finding could be a useful addition to the knowledge base in the literature concerning CSMA/CA from a purely analytical perspective, and may act as a starting point for arriving at the exact expression for the idle period for CSMA/CA with multiple backoff stages. It may also be useful in spectrum sensing or wireless sniffing applications where there is a need to detect CSMA/CA transmissions without the full physical transceiver implementation.

References

[1] IEEE Std. 802.11-2007, Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications, IEEE, 2007.

[2] IEEE Std. 802.15.4, Part 15.4: Wireless Medium Access Control (MAC) and Physical Layer (PHY) specifications for Low-Rate Wireless Personal Area Networks (LR-WPANs), IEEE, 2003.

[3] R.J. Jayabal, C.T. Lau, Throughput analysis of PD-CSMA/CA in two interference-limited cells, in: Proc. International Symposium on Computer, Consumer and Control, IS3C, 2012 (in press).

[4] T. Sakurai, Hai L. Vu, MAC access delay of IEEE 802.11 DCF, IEEE Transactions on Wireless Communications 6 (5) (2007) 1702–1710.

[5] Y. Li, C. Wang, K. Long, W. Zhao, Modeling channel access delay and jitter of IEEE802.11 DCF, Wireless Personal Communications 47 (3) (2008) 417–440.

[6] Qian Dong, Waltenegus Dargie, Analysis of collision probability in unsaturated situation, in: Proc. ACM Symposium on Applied Computing, SAC, 2010, pp. 772–777.

[7] G. Bianchi, IEEE 802.11-saturation throughput analysis, IEEE Communications Letters 2 (12) (1998) 318–320.
[8] G. Bianchi, Performance analysis of the IEEE 802.11 distributed coordination function, IEEE Journal on Selected Areas in Communications 18 (3) (2000) 535–547.
[9] E. Ziouva, T. Antonakopoulos, CSMA/CA performance under high traffic conditions: throughput and delay analysis, Computer Communications 25 (3) (2002) 313–321.
[10] C.H. Foh, J.W. Tantra, Comments on IEEE 802.11 saturation throughput analysis with freezing of backoff counters, IEEE Communications Letters 9 (2) (2005) 130–132.
[11] C. Hu, H. Kim, J.C. Hou, An analysis of the binary exponential backoff algorithm in distributed MAC protocols, Technical Report No. UIUCDCS-R-2005-2599 (UILU-ENG-2005-1794), University of Illinois, Urbana Champaign, 2005.
[12] R.J. Jayabal, C.T. Lau, Deriving the suspended backoff counter distribution in CSMA/CA, International Journal of Computer Applications 28 (9) (2011) 14–20.
[13] R.J. Jayabal, C.T. Lau, Exact expression for single-staged CSMA/CA's idle period distribution, in: Proc. International Symposium on Computer, Consumer and Control, 2012 (in press).
[14] R. Bowden, A. Coyle, Idle times analysis in a CSMA/CA network, in: Proc. IEEE International Conference on Networks, 2007, pp. 443–448.
[15] The network simulator ns-2: http://www.isi.edu/nsnam/ns/.