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A multivariate Poisson-lognormal regression model for prediction of crash counts by severity, using Bayesian methods

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Abstract

Numerous efforts have been devoted to investigating crash occurrence as related to roadway design features, environmental factors and traffic conditions. However, most of the research has relied on univariate count models; that is, traffic crash counts at different levels of severity are estimated separately, which may neglect shared information in unobserved error terms, reduce efficiency in parameter estimates, and lead to potential biases in sample databases. This paper offers a multivariate Poisson-lognormal (MVPLN) specification that simultaneously models crash counts by injury severity. The MVPLN specification allows for a more general correlation structure as well as overdispersion. This approach addresses several questions that are difficult to answer when estimating crash counts separately. Thanks to recent advances in crash modeling and Bayesian statistics, parameter estimation is done within the Bayesian paradigm, using a Gibbs Sampler and the Metropolis–Hastings (M–H) algorithms for crashes on Washington State rural two-lane highways. Estimation results from the MVPLN approach show statistically significant correlations between crash counts at different levels of injury severity. The non-zero diagonal elements suggest overdispersion in crash counts at all levels of severity. The results lend themselves to several recommendations for highway safety treatments and design policies. For example, wide lanes and shoulders are key for reducing crash frequencies, as are longer vertical curves.

Keywords: Bayesian inference; Bayes' theorem; Crash severity; Gibbs sampler; Highway safety; Metropolis–Hastings algorithm; Markov chain Monte Carlo (MCMC) simulation; Multivariate Poisson-lognormal regression

1. Introduction

Roadway safety is a major concern for the general public and public agencies. Roadway crashes claim many lives and cause substantial economic losses each year. In the U.S. traffic crashes bring about more loss of human life (as measured in human-years) than almost any other cause—falling behind only cancer and heart disease (NHTSA, 2005). The situation is of particular interest on rural two-lane roadways, which experience significantly higher fatality rates than urban roads. The annual cost of traffic crashes is estimated to be $231 billion, or $820 per capita in 2000 (Blincoe et al., 2002). These costs do not include the cost of delays imposed on other travelers, which also are significant, particularly when crashes occur on busy roadways. Schrank and Lomax (2002) estimate that over half of all traffic delays are due to non-recurring events, such as crashes, costing on the order of $1000 per peak-period driver per year, particularly in urban areas. Thus, while vehicle and roadway design are improving, and growing congestion may be reducing impact speeds, crashes are becoming more critical in many ways, particularly in societies that continue to motorize.

Given the importance of roadway safety, there has been considerable crash prediction research (see, e.g., Hauer, 1986, 1997, 2001; Abdel-Aty and Radwan, 2000; Ulfarsson and Shankar, 2003; Kweon and Kockelman, 2005; Lord and Persaud, 2000; Lord et al., 2005; Ma and Kockelman, 2006a,b; Karlaftis and Rarko, 1998; Shankar et al., 1998; Khattak et al., 2006). Crash frequencies are commonly collected by severity on relatively homogenous roadway segments, supporting the development of crash count models. However, such research has relied on univariate count models; that is, traffic crash counts at different levels of severity are estimated separately. The widely used uni-

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variate count data models ignore the following issues: interdependence due to latent factors is likely to exist across crash rates at different levels of severity for a specific segment of roadway. Recently, Ma and Kockelman (2006a,b) applied a multivariate Poisson (MVP) specification to model crash counts at different levels of severity simultaneously. However, this MVP specification allows only for a common added Poisson error term, resulting in equal positive correlations across crash counts and a very specific data pattern where all counts are equally shifted. In addition, this MVP specification does not allow for overdispersion.

Using a multivariate Poisson-lognormal (MVPLN) specification, as well as Bayesian estimation techniques, this work models correlated traffic crash counts simultaneously at different levels of severity. The MVPLN specification allows for a more general correlation structure as well as overdispersion. This approach addresses several questions that are difficult to answer by estimating them separately. With recent advances in crash modeling and Bayesian statistics, parameter estimation is done within the Bayesian paradigm, using a Gibbs Sampler and the Metropolis–Hastings (M–H) algorithms. The data come from Washington State rural two-lane highways in 2002, using the Highway Safety Information System (HSIS) database. The results lend themselves to recommendations for highway safety treatments and general design policies.

This paper is organized as follows: related research studies are reviewed first. The model's formulation and data sets are then discussed, followed by estimation results, concluding remarks, and future research directions.

2. Literature review

Models of crash (or injury) counts can be classified into two major streams: (1) the conventional univariate Poisson and related models, such as the negative binomial (NB); (2) potentially more realistic specifications, such as the MVP and MVPLN. The first stream has provided a means for investigating associations between crash frequency and many crucial factors, such as traffic volume, access density, posted speed limit and number of lanes (see, e.g., Miaou et al., 1993; Miaou and Lum, 1993; Miaou, 1994, 1996, 2001; Fridstrøm et al., 1995; Johansson, 1996; Vogt and Bared, 1998; Vogt, 1999; Balkin and Ord, 2001; Zegeer et al., 2002; Pernia et al., 2004). There also has been considerable interest in models that allow for excessive zeros, such as zero-inflated Poisson (ZIP) and zero-inflated negative binomial (ZINB) regression approaches (see, e.g., Lord et al., 2005; Shankar et al., 1997; Garber and Wu, 2001; Lee and Mannering, 2002; Kumara and Chin, 2003; Miaou and Lord, 2003; Rodriguez et al., 2003; Shankar et al., 2003; Noland and Quddus, 2004; Qin et al., 2004).

Due to computational and statistical advances, panel data (in which a cross-section of segments, intersections, etc. is observed over time) have become more amenable to rigorous analysis. In traffic crash analyses, there are a great many unobserved explanatory variables that affect frequencies and severities. Panel data can be used to deal with heterogeneity among individuals. To address the heterogeneity, many recent studies have used (univariate) panel count data models, such as random-effect negative binomial (RENB) and fixed-effect negative binomial (FENB) regression models (Kweon and Kockelman, 2005; Karlaftis and Rarko, 1998; Shankar et al., 1998; Chin and Quddus, 2003).

Such past research endeavors, however, have neglected the role of unobserved factors across different types of counts (e.g., the number of fatalities and the number of disabling injuries). Recognizing the need for such considerations, Ladron de Guevara and Washington (2004) investigated the simultaneity of fatality and injury crash outcomes. Bijleveld (2005) also examined the correlation structure between crash and injury counts. As expected, he found significant correlations. However, he did not control for any covariates. Multivariate models (of count data), like Ma and Kockelman's MVP (2006a,b) or Li et al's MVZIP (1999), can help correct for this.

This work models correlated traffic crash counts simultaneously at different levels of severity using an MVPLN specification, allowing for a very general correlation structure as well as overdispersion. Such specifications are challenging to estimate. Karlis (2003) developed an EM algorithm for an MVP model, and Ma and Kockelman (2006a,b) used Gibbs sampling, as well as Metropolis–Hastings algorithms, within an MCMC simulation framework.

In recent years, Bayesian methods have found several applications in traffic crash analysis. Christiansen et al. (1992) and MacNab (2003) developed hierarchical Poisson models for crash counts and surveillance data. Miaou and Song (2005) developed a Bayesian multivariate spatial generalized linear mixed model (GLMM) to rank sites for safety improvements using Texas' county-level crash data. And Liu et al. (2005) used a hierarchical Bayesian framework to estimate ZIP regression models and develop safety performance functions (SPFs) for two-lane highways. Pawlovich et al. (2006) employed a Bayesian approach to assess impacts of road design measures on crash frequencies and rates. And Washington and Oh (2006) developed a Bayesian methodology for incorporating expert judgment in ranking countermeasure effectiveness under uncertainty.

Bayesian estimation methods generate a multivariate posterior distribution across all parameters of interest, as opposed to the traditional maximum likelihood estimation approach, which emphasizes and offers only the modal values of parameters (and relies on asymptotic properties to ascertain covariance).

This paper introduces an MVPLN approach to simultaneously model crash counts by injury severity. A Gibbs sampler and a Metropolis–Hastings (M–H) algorithm are used to estimate the parameters of interest using Bayesian methods. For comparison purposes, a series of independent (univariate) Poisson models for injury counts also are estimated.

3. Model structure and estimation

3.1. Mathematical formulation

Univariate Poisson regression models cannot account for correlations for different levels of severity; instead, one needs multivariate count data models. Crash counts (across severities) effectively require a multivariate model because the response

is multi-dimensional. Moreover, in practice, omitted variables (such as driveway density and sight distances) may simultaneously affect all crash counts at different levels of severity for a particular roadway segment, thus introducing correlation. Several such models have been developed (see, e.g., Karlis, 2003; Arbous and Kerrich, 1951; King, 1989; Winkelmann, 2000; Kockelman, 2001; Tsionas, 2001). However, these specifications support only a common unobserved error term among counts.

Here, the focus is placed on the correlated counts within individual roadway segments. Crash counts across roadway segments are assumed to be independent (e.g., there is no spatial correlation[3]). The variance-covariance matrix of $\mathbf{y}$ can be expressed as below:

$$\mathrm{Var}(\mathbf{y}_{nS\times 1}) = \begin{bmatrix} \Omega_1 & 0 & \cdots & 0 \\ 0 & \Omega_2 & \cdots & 0 \\ & & \vdots & \\ 0 & 0 & \cdots & \Omega_n \end{bmatrix} \tag{1}$$

where

$$\Omega_i = \begin{bmatrix} \omega_{11}^i & \omega_{12}^i & \cdots & \omega_{1S}^i \\ \omega_{21}^i & \omega_{22}^i & \cdots & \omega_{2S}^i \\ & & \vdots & \\ \omega_{S1}^i & \omega_{S2}^i & \cdots & \omega_{SS}^i \end{bmatrix} \quad \text{for} \quad i = 1, 2, \ldots, n \tag{2}$$

Let $\vec{\varepsilon}_i = (\varepsilon_{i1}, \varepsilon_{i2}, \ldots \varepsilon_{iS})'$ denote the severity-level-specific unobserved heterogeneity for roadway segment i [$i = 1, 2, \ldots, n$, where n is the number of roadway segments], s denote the severity level [$s = 1, 2, \ldots, S$, where S is the number of severity levels], and $\varepsilon = (\vec{\varepsilon'}_1, \vec{\varepsilon'}_1, \ldots, \vec{\varepsilon'}_n)'$ denote the severity-level-specific unobserved heterogeneity across roadway segments.

Assume that crash counts y_{is}, conditioned on $\vec{\varepsilon}_i$, the severity-level-specific explanatory variables x'_{is} and their coefficients of β_s, are independent Poisson distributed.

$$y_{is} \,|\vec{\varepsilon}_i, \beta_s\,, x_{is} \sim \mathrm{Poisson}(\lambda_{is}) \tag{3}$$

where $\lambda_{is} = \exp(x'_{is}\beta_s + \varepsilon_{is})$. The unobserved heterogeneity terms $\vec{\varepsilon}_i$ are assumed to be uncorrelated with the control (i.e., explanatory) variables.

[3] In reality, spatial correlation may exist and be significant. For example, zoning and design policies create correlation across sites within a city; access management and other policies may simply shift the location of certain crash types. The former leads to positive correlation, the latter to negative. Song et al. (2006) developed Bayesian multivariate spatial models to investigate spatial correlations in four types of crashes using Texas' county-level data.

Let $\Lambda_i = \mathrm{diag}(\vec{\lambda}_i)$. This is an $S \times S$ matrix, where $\vec{\lambda}_i = (\lambda_{i1}, \lambda_{i2}, \ldots, \lambda_{iS})$ and $\lambda_{is} = \xi_{is}u_{is}$. Let $\vec{u}_i = \exp(\vec{\varepsilon}_i)$, where $\vec{u}_i = (u_{i1}, u_{i2}, \ldots, u_{iS})'$. Conditioning on β and Σ, the mean and covariance matrix of the marginal distribution of $\vec{y}_i$ can be obtained as follows (Greene, 2003):

$$\begin{aligned} E(\vec{y}_i\,|\beta, x_i, \Sigma) &= E_{\vec{u}_i}(E_{\vec{y}_i|\vec{u}_i}(\vec{y}_i\,|\beta, x_i, \vec{u}_i, \Sigma)) \\ &= E_{\vec{u}_i}(\mathrm{diag}(\vec{\xi}_i)\vec{u}_i) = \vec{\lambda}_i \end{aligned} \tag{4}$$

$$\begin{aligned} \mathrm{Var}(\vec{y}_i\,|\beta, x_i, \Sigma) &= E_{\vec{u}_i}(\mathrm{Var}_{\vec{y}_i|\vec{u}_i}(\vec{y}_i\,|\beta, x_i, \vec{u}_i, \Sigma)) \\ &\quad + \mathrm{Var}_{\vec{u}_i}(E_{\vec{y}_i|\vec{u}_i}(\vec{y}_i\,|\beta, x_i, \vec{u}_i, \Sigma)) \\ &= E_{\vec{u}_i}(\mathrm{diag}(\mathrm{diag}(\vec{\xi}_i)\vec{u}_i)) + \mathrm{Var}_{\vec{u}_i}(\mathrm{diag}(\vec{\xi}_i)\vec{u}_i) \\ &= \Lambda_i + \Lambda_i[\exp(\Sigma) - 11']\Lambda_i \end{aligned} \tag{5}$$

where $\beta = (\beta_1, \beta_2, \ldots, \beta_S)'$, $x_i = (x_{i1}, x_{i2}, \ldots, x_{iS})'$, $\vec{\xi}_i = (\xi_{i1}, \xi_{i2}, \ldots, \xi_{iS})'$, $11'$ is an identity matrix of dimension S, and $\Sigma = \begin{Bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1S} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2S} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{S1} & \sigma_{S2} & \cdots & \sigma_{SS} \end{Bmatrix}$. The length of β is $k = k_1 + k_2 + \ldots + k_S$, where k_s is the length of β_s.

From Eq. (5), the variance–covariance terms, across counts, can be obtained as follows:

$$\begin{aligned} \mathrm{Cov}(y_{is}, y_{il}) &= 0 + \lambda_{is}[\exp(\sigma_{sl}) - 1]\lambda_{il} \\ &= \xi_{is}\exp(\sigma_{ss}/2)[\exp(\sigma_{sl}) - 1]\xi_{il}\exp(\sigma_{ll}/2), \\ \text{for} \quad s \neq l \mathrm{Var}(y_{is}, y_{is}) &= \lambda_{is} + \lambda_{is}[\exp(\sigma_{ss}) - 1]\lambda_{is} \end{aligned} \tag{6}$$

The correlation between crash counts within segments is obtained as follows:

$$\begin{aligned} \mathrm{Corr}(y_{is}, y_{il}) &= \frac{\xi_{is}[\exp(\sigma_{sl}) - 1]\xi_{il}}{\sqrt{\xi_{is}\exp(-\sigma_{ss}/2) + \xi_{is}^2[\exp(\sigma_{ss}) - 1]}\sqrt{\xi_{il}\exp(\sigma_{ll}/2) + \xi_{il}^2[\exp(\sigma_{ll}) - 1]}} \\ &= \frac{\exp(\sigma_{sl}) - 1}{\sqrt{\xi_{is}^{-1}\exp(-\sigma_{ss}/2) + \exp(\sigma_{ss}) - 1}\sqrt{\xi_{il}^{-1}\exp(-\sigma_{ll}/2) + \exp(\sigma_{ll}) - 1}} \end{aligned} \tag{7}$$

where $s \neq l$.

This correlation is unrestricted and can be positive or negative, depending on the sign of σ_{sl}, the (s,l) element of Σ. Moreover, this specification implies overdispersion[4], since $\sigma_{ss} > 0$ for $s = 1, 2, \ldots, S$.

Based on Eq. (3), the likelihood of observation i can be represented by the following equation:

$$P(\vec{y}_i\,|\vec{\varepsilon}_i, \beta, x_i) = \prod_{s=1}^{S} f_{\mathrm{Poisson}}(y_{is}\,|\lambda_{is}) \tag{8}$$

where $\lambda_{is} = \xi_{is}u_{is} = \exp(x'_{is}\beta_s + \varepsilon_{is})$.

[4] Overdispersion refers to the situation in which variance is greater than mean.

Unfortunately, the *marginal* distribution of the crash counts $\vec{y}_i$ cannot be obtained by direct computation. Obtaining the marginal distribution requires the evaluation of an S-variate integral of the Poisson distribution with respect to the distribution of $\vec{\varepsilon}_i$,

$$P(\vec{y}_i \,|\vec{\lambda}_i, \Sigma) = \int \prod_{s=1}^{S} f_{\text{Poisson}}(y_{is} \,|x_{is}, \beta_s, \varepsilon_{is}) \phi_S[\vec{\varepsilon}_i \,|0, \Sigma] d\vec{\varepsilon}_i \tag{9}$$

where ϕ_s is the S-variate normal distribution. This S-dimensional integral cannot be algebraically implemented in closed form for arbitrary Σ.

3.2. Estimating parameters via MCMC

In order to illuminate crash rate relationships, the MVPLN model's unknown parameters need to be estimated. Chib et al. (1998) showed how to estimate a posterior distribution of unknown parameters for their models of panel count data[5], and Plassmann and Tideman (2001) developed a Gibbs sampler to estimate parameters in a univariate Poisson-lognormal model.

Based on Press (1982) and Gelman et al. (2004), the Wishart distribution is commonly used as a conjugate prior for the inverse of variance-covariance parameters. According to Press (1982), the Wishart and normal distributions are very helpful for multivariate analysis. Suppose that the parameters (β, Σ) independently have the prior distributions:

$$\beta \sim \phi_k(\beta_0, V_{\beta_0}), \quad \Sigma^{-1} \sim f_W(\nu_\Sigma, V_\Sigma) \tag{10}$$

where $\beta_0 = (\beta_{01}, \beta_{02}, \ldots, \beta_{0S})'$, $V_{\beta_0} = \begin{bmatrix} V_{\beta_{01}} & 0 & \cdots & 0 \\ 0 & V_{\beta_{02}} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & V_{\beta_{0S}} \end{bmatrix}$, f_W (.,.) is the Wishart distribution with ν_Σ degrees of freedom and scale matrix V_Σ, and β_0, $V_{\beta 0}$, ν_Σ and V_Σ are known hyperparameters. The prior distribution for β_s can written as $\beta_s \sim \phi_{k_s}(\beta_{0s}, V_{\beta_{0s}})$ for $s = 1, 2, \ldots, S$.

According to Bayes' theorem (*posterior* $\propto$ *prior* $\times$ *likelihood*), the posterior kernel can be written as follows:

$$\pi(\Sigma, \beta \,|y, X) \propto \phi_k(\beta_0, V_{\beta_0}) f_W(\nu_\Sigma, V_\Sigma) \prod_{i=1}^{n} \int \prod_{s=1}^{S} f_{\text{Poisson}}(y_{is} \,|x_{is}, \beta_s, \varepsilon_{is}) \phi_S(\vec{\varepsilon}_i \,|0, \Sigma) \mathrm{d}\vec{\varepsilon}_i$$

Using data augmentation[6], the latent effects ε can be thought of as ("nuisance") parameters to be estimated. Therefore, the joint posterior density of Σ, ε, and β is written as follows:

$$\pi(\Sigma, \varepsilon, \beta \,|y, X) \propto \phi_k(\beta_0, V_{\beta_0}) f_W(\nu_\Sigma, V_\Sigma) \prod_{i=1}^{n} \prod_{s=1}^{S} f_{\text{Poisson}}(y_{is} \,|x_{is}, \beta_s, \varepsilon_{is}) \phi_S(\vec{\varepsilon}_i \,|0, \Sigma) \tag{11}$$

Thanks to this technique, the parameters can be "blocked" as Σ, ε, and β, after which the joint posterior is simulated by iteratively sampling from the following three conditional distributions: $\pi^p[\Sigma^{-1}|\varepsilon]$, $\pi^p[\varepsilon|y, X, \beta, \Sigma]$, and $\pi^p[\beta|y, X, \varepsilon, \Sigma]$, where $\pi^p(|)$ denotes the posterior conditional density function.

The draws are sampled sequentially using the most recent values of the conditioning variables at each step.

3.3. Gibbs sampler with embedded M–H algorithms

After manipulating the posterior equation (11), the posterior of Σ^{-1} conditional on data and other parameters can be written as

$$\pi(\Sigma^{-1} \,|\boldsymbol{\varepsilon}) \propto f_W(\Sigma^{-1} \,|\nu_\Sigma, V_\Sigma) \prod_{i=1}^{n} \phi_S(\vec{\varepsilon}_i|0, \boldsymbol{\Sigma}) \tag{12}$$

where f_W denotes the Wishart density with ν_Σ degrees of freedom and scale matrix V_Σ.

After manipulating Eq. (12), this density can be written as a Wishart kernel with degrees of freedom $n + \nu_\Sigma$ and scale matrix $\left[V_\Sigma^{-1} + \sum_{i=1}^{n} (\vec{\varepsilon}_i \vec{\varepsilon}')_i\right]^{-1}$. In other words,

$$\Sigma^{-1} \,|\varepsilon \sim f_W \left(n + \nu_\Sigma, \left[V_\Sigma^{-1} + \sum_{i=1}^{n} (\vec{\varepsilon}_i \vec{\varepsilon}')_i\right]^{-1}\right) \tag{13}$$

This is a known parametric distribution and thus can be sampled using a Gibbs sampler.

In order to sample ε from its posterior density $\pi(\varepsilon|y, \beta, \Sigma) = \prod_{i=1}^{n} \pi(\vec{\varepsilon}_i|\vec{y}_i, \beta, \Sigma)$, consider simply the ith posterior kernel density of $\vec{\varepsilon}_i$, due to an assumption of no spatial correlation across segments.

$$\pi(\vec{\varepsilon}_i|\vec{y}_i, x_i, \beta, \Sigma) = C_i \phi_S(\vec{\varepsilon}_i \,|\Sigma) \prod_{s=1}^{S} \exp(-\lambda_{is}) \lambda_{is}^{y_{is}} = C_i \pi^{\mathrm{p}}(\vec{\varepsilon}_i|\vec{y}_i, x_i, \beta, \Sigma), \tag{14}$$

where $\lambda_{is} = \exp(x'_{is}\beta_s + \varepsilon_{is})$. Draws from this conditional density can be obtained by developing an M–H algorithm, as described below.

Following Chib et al. (1998), the multivariate t distribution is used as the proposal density. Let $\hat{\vec{\varepsilon}}_i = \underset{\vec{\varepsilon}_i}{arg\max}[\ln \pi^p(\vec{\varepsilon}_i \,|\vec{y}_i, x_i, \beta, \Sigma)]$ and $V_{\varepsilon_i} = (-H_{\varepsilon_i})^{-1}$ be the inverse of the Hessian of $\ln \pi^{\mathrm{p}}(\vec{\varepsilon}_i \,|\vec{y}_i, x_i, \beta, \Sigma)$ at the mode $\hat{\vec{\varepsilon}}_i$. The mode $\hat{\vec{\varepsilon}}_i$ and variance–covariance matrix V_{ε_i} can be obtained using the Newton–Raphson algorithm with

[5] Estimation of β in the panel count data models is similar to estimation of β_s in the MVPLN model.

[6] Data augmentation views unobserved or latent variables as unknown parameters (to be estimated), in order to establish iterative algorithms.

the gradient vector $\vec{g}_{\varepsilon_i} = -\Sigma^{-1}\vec{\varepsilon}_i + \left[\vec{y}_i - \exp\left(x_i\beta + \vec{\varepsilon}_i\right)\right]$ and Hessian matrix $H_{\varepsilon_i} = -\Sigma^{-1} - \mathrm{diag}[\exp(x_i\beta + \vec{\varepsilon}_i)]$, where $x_i = \begin{bmatrix} x'_{i1} & 0 & \dots & 0 \\ 0 & x'_{i2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & x'_{iS} \end{bmatrix}$ and $\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_S \end{bmatrix}$. Then, the proposal density is given by $f_T(\vec{\varepsilon}_i \,|\hat{\varepsilon}_i, V_{\varepsilon_i}, \nu_\varepsilon)$, a multivariate-$t$ distribution with ν_ε degrees of freedom (where ν_ε can be used as a tuning parameter in the M–H algorithms to make sure that the acceptance rate[7] lies between 20 and 45%[8]). A proposal value $\vec{\varepsilon}_i^*$ is drawn from $f_T(\vec{\varepsilon}_i \,|\hat{\varepsilon}_i, V_{\varepsilon_i}, \nu_\varepsilon)$, and the chain moves to $\vec{\varepsilon}_i^*$ from the current point $\vec{\varepsilon}_i$ with probability.

$$\alpha(\vec{\varepsilon}_i, \vec{\varepsilon}_i^* \,|\vec{y}_i, x_i, \beta, \Sigma) = \min\left\{\frac{\pi^p(\vec{\varepsilon}_i^* \,|\vec{y}_i, x_i, \beta, \Sigma) f_T(\vec{\varepsilon}_i \,|\hat{\varepsilon}_i, V_{\varepsilon_i}, \nu_\varepsilon)}{\pi^p(\vec{\varepsilon}_i \,|\vec{y}_i, x_i, \beta, \Sigma) f_T(\vec{\varepsilon}_i^* \,|\hat{\varepsilon}_i, V_{\varepsilon_i}, \nu_\varepsilon)}, 1\right\} \quad (15)$$

If $\alpha(\vec{\varepsilon}_i, \vec{\varepsilon}_i^* \,|\vec{y}_i, x_i, \beta, \Sigma)$ is greater than U (where U is uniformly distributed on [0,1]), the proposal value $\vec{\varepsilon}_i^*$ is accepted; otherwise, the current value $\vec{\varepsilon}_i$ is kept as the new draw for the Markov chain.

The samples of β_s, conditional on ε, y, X, Σ, and, β_{-s} (where β_{-s}=[β_1, β_2, …, β_{s-1}, β_{s+1}, …, β_s]) are drawn from the posterior distribution, which is proportional to:

$$\begin{aligned}
&\pi^p(\beta_s \,|y, X, \varepsilon, \Sigma) \\
&= \pi^p(\beta_s \,|y_{\cdot s}, X, \varepsilon_{\cdot s}, \Sigma) \prod_{j=1, j\neq s}^{S} \pi^p(\beta_j \,|y_{\cdot j}, X, \varepsilon_{\cdot j}, \Sigma) \\
&= C_{-s}\pi^p(\beta_s \,|y_{\cdot s}, X, \varepsilon_{\cdot s}, \Sigma) \propto \phi_{k_s}(\beta_s \,|\beta_{0s}, V_{\beta_{0s}}) \\
&\quad \prod_{i=1}^{n} \exp[-\exp(x'_{is}\beta_s + \varepsilon_{is})][\exp(x'_{is}\beta_s \\
&\quad + \varepsilon_{is})]^{y_{is}} \propto \phi_{k_s}(\beta_s \,|\beta_{0s}, V_{\beta_{0s}}) p(y_{\cdot s} \,|\beta_s, \varepsilon_{\cdot s}) \quad (16)
\end{aligned}$$

where $C_{-s} = \prod_{j=1, j\neq s}^{S} \pi^p(\beta_j \,|y_{\cdot j}, X, \varepsilon_{\cdot j}, \Sigma)$ (which does not involve β_s and thus serves as a constant), and $p(y_{\cdot s} \,|X, \beta_s, \varepsilon_{\cdot s}) = \prod_{i=1}^{n} \exp[-\exp(x'_{is}\beta + \varepsilon_{is})][\exp(x'_{is}\beta + \varepsilon_{is})]^{y_{is}}$ is the probability mass function of $y_{\cdot s}$=(y_{1s}, y_{2s}, …, y_{ns}) given β_s, X and $\varepsilon_{\cdot s}$ = (ε_{1s}, ε_{2s}, …, ε_{ns}). Note that the β_s's ($s\in\{1,2,\dots,S\}$) are assumed to be independent of one another.

A scheme similar to the one sampling $\vec{\varepsilon}_i$ is developed here to sample β_s. The multivariate-t once again serves as the proposal density. Let $\hat{\beta}_s = \underset{\beta_s}{arg\max}[\ln \pi^p(\beta_s \,|y_{\cdot s}, X, \varepsilon_{\cdot s}, \beta_{-s}, \Sigma)]$ be the mode, and $V_{\beta_s} = (-H_{\beta_s})^{-1}$ the inverse of the Hessian of $\ln \pi^p(\beta_s|y,X,\boldsymbol{\varepsilon}, \beta_{-s}, \Sigma)$ at the mode $\hat{\beta}_s$. The mode $\hat{\beta}_s$ and variance–covariance matrix V_{β_s} can be obtained using the Newton–Raphson algorithm with the gradient vector $\vec{g}_{\beta_s} = -V_{\beta_{0s}}^{-1}(\beta_s - \beta_{0s}) + \sum_{i=1}^{n}[y_{is} - \exp(x'_{is}\beta_s + \varepsilon_{is})]x_{is}$ and Hessian matrix $H_{\beta_s} = -V_{\beta_{0s}}^{-1} - \sum_{i=1}^{n}[\exp(x'_{is}\beta_s + \vec{\varepsilon}_{is})]x_{is}x'_{is}$. Then, the proposal density is given by $f_T(\beta_s \,|\hat{\beta}_s, V_{\beta_s}, \nu_\beta)$, a multivariate-$t$ distribution with ν_β degrees of freedom (where ν_β can be used as a tuning parameter in the M–H algorithms to make sure that the acceptance rate lies between 20 and 45%). A proposal value β_s^* is drawn from $f_T(\beta_s \,|\hat{\beta}_s, V_{\beta_s}, \nu_\beta)$, and the chain moves to β_s^* from the current point β_s with probability:

$$\alpha(\beta_s, \beta_s^* \,|y, X, \varepsilon, \beta_{-s}, \Sigma) \min\left\{\frac{\pi^p(\beta_s^* \,|y, X, \varepsilon, \beta_{-s}, \Sigma) f_T(\beta_s \,|\hat{\beta}_s, V_{\beta_s}, \nu_\beta)}{\pi^p(\beta_s \,|y, X, \varepsilon, \beta_{-s}, \Sigma) f_T(\beta_s^* \,|\hat{\beta}_s, V_{\beta_s}, \nu_\beta)}, 1\right\} \quad (17)$$

If $\alpha(\beta_s, \beta_s^* \,|y, X, \varepsilon, \beta_{-s}, \Sigma)$ is greater than U (where U is uniformly distributed on [0,1]), the proposal value β_s^* is accepted; otherwise, the current value β_s is kept as the new draws for the Markov chain.

4. Data description

The crash data sets used here were collected from Washington State through the Highway Safety Information System (HSIS). In order to examine traffic crashes patterns on rural two-lane roadways, this research considers crashes in the Puget Sound region. A random sample of 60% of all rural two-lane road segments in this region was used for model estimation. A total of 7773 rural two-lane highway segments (with an average segment length of 0.0655 miles[9] and a total of 510 miles) are available for analysis. This sample contains 16 fatal crashes, 50 disabling-injury crashes, 180 non-disabling-injury crashes, 175 possible-injury crashes and 532 property-damage-only (PDO). Table 1 reports summary statistics for the dependent and independent variables employed in the analysis. A variety of readily available variables are controlled for in the model, including design features, traffic intensity, location information, and roadway functional classification.

[7] The acceptance rate is the fraction of proposed samples that is accepted. If the proposal steps are too small, the chain will move around the space slowly and thus converge slowly on the true posterior density. If the proposal steps are too large, the acceptance rate will be very low because the proposals are likely to land in regions of much lower probability density.

[8] Chib and Greenberg (1995) believe that an acceptance rate of 23% is desirable as the number of dimensions approaches infinity, and an acceptance rate of 45 percent is desirable for a one-dimensional random-walk chain.

[9] It is quite possible that very short segments do not faithfully represent the actual location of crashes, since police officers may locate crashes only to the nearest tenth of a mile. Cluster analysis, wherein similar segments/conditions are merged (providing higher crash counts) can address some of this bias in reporting. Ma and Kockelman (2006a,b) conducted such an analysis with Washington State data.

Table 1
Summary statistics of variables

Variable name	Mean	Standard deviation	Min.	Max.
Dependent variables				
Number of fatal crashes	0.002058	.04533	0	1
Number of disabling injury crashes	0.006433	.07995	0	1
Number of non-disabling injury crashes	0.02316	.1587	0	3
Number of possible injury crashes	0.02251	.2045	0	11
Number of PDO crashes	0.06844	.3345	0	12
Independent variables				
Segment length (miles)	0.0655	.08689	.00	1.92
Horizontal curve length (ft)	247.6	475.4	.00	4715
Degree of curvature (°/100 ft)	2.337	5.462	.00	100.5
Vertical curve length (ft)	302.7	376.0	.00	3200
Vertical grade (%)	1.805	1.991	.00	16.13
Average shoulder width on each side (ft)	2.087	1.298	.00	16.50
Surface width (ft)[a]	24.00	4.461	16.0	73.0
Posted speed limit (miles/h)	49.62	8.163	25.0	60.0
Posted speed limit squared (miles^2/h^2)	2528	715.5	625	3600
Average annual daily traffic (AADT)	3757	2729	254	28,624
Indicator for principal arterial: 1 = yes, 0 = otherwise	0.48	0.499	0	1
Indicator for minor arterial: 1 = yes, 0 = otherwise	0.28	0.451	0	1
Indicator for collector: 1 = yes, 0 = otherwise	0.24	0.430	0	1
Indicator for level terrain: 1 = yes, 0 = otherwise	0.36	0.482	0	1
Indicator for rolling terrain: 1 = yes, 0 = otherwise	0.60	0.491	0	1
Indicator for mountainous terrain: 1 = yes, 0 = otherwise	0.04	0.194	0	1
Vehicle miles traveled (VMT) in 2002	88,106	142,830	.00	2,679,710
The natural logarithm of VMT	10.45	2.737	−22.35	14.80
Number of observations		7773		

[a] Surface width does not include the width of shoulders (paved or unpaved).

5. Model estimation and results

5.1. Model estimation

The MVPLN regression model was estimated using a Bayesian approach. The starting values for β came from distinct univariate Poisson models (using the method of maximum likelihood estimation (MLE)). The starting values for Σ are $I_5 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$. The MLE estimates for the five univariate Poisson models can be found in Ma (2006). A Gibbs sampler and two M–H algorithms were coded in the R language

Table 2
PDO crash frequency MVPLN model results

Variable definition	Mean	Standard error	The 95% (2.5–97.5%) sample-based credible sets	
Constant	−12.64	0.4562	−13.38	−11.88
Horizontal curve length (ft)	*2.09E−05*	*1.35E−05*	*−1.31E−06*	*4.27E−05*
Degree of curvature (°/100 ft)	0.1241	6.31E-03	0.1136	0.1344
Vertical curve length (ft)	−2.05E−04	1.97E−05	−2.37E−04	−1.73E−04
Vertical grade (%)	0.1377	0.01441	0.1134	0.1609
Average shoulder width (ft)	−0.01125	3.54E−03	−0.01694	−5.28E−03
Surface width (ft)	−0.01520	5.25E−04	−0.01607	−0.01434
Posted speed limit (miles/h)	0.01493	2.89E-03	0.01014	0.01972
Posted speed limit squared (miles^2/h^2)	−1.53E−04	8.64E−05	−2.97E−04	−1.33E−05
Average annual daily traffic (AADT)	4.79E−05	2.03E−06	4.46E−05	5.13E−05
Indicator for minor arterial: 1 = yes, 0 = otherwise	*−0.01112*	*0.01631*	*−0.03759*	*0.01568*
Indicator for collector: 1 = yes, 0 = otherwise	*−0.009441*	*0.01872*	*−0.04049*	*0.02080*
Indicator for rolling terrain: 1 = yes, 0 = otherwise	0.03929	0.01439	0.01526	0.06240
Indicator for mountainous terrain: 1 = yes, 0 = otherwise	0.6120	0.04687	0.5355	0.6888
Number of observations		7773		

Note: Italics are used for parameters that do not differ from zero in a statistically significant way, based on the 95% (2.5–97.5) sample-based credible sets.

Table 3
Possible-injury crash frequency MVPLN model results

Variable definition	Mean	Standard error.	The 95% (2.5–97.5%) sample-based credible sets	
Constant	−15.85	0.8120	−17.22	−14.53
Horizontal curve length (ft)	*2.90E−05*	*2.37E−05*	*−8.46E−06*	*6.90E−05*
Degree of curvature (°/100ft)	0.1031	7.09E-03	0.09136	0.1147
Vertical curve length (ft)	−2.97E−04	1.30E−05	−3.18E−04	−2.76E−04
Vertical grade (%)	0.1616	9.20E-03	0.1465	0.1766
Average shoulder width (ft)	−8.71E−03	9.48E−04	−0.01027	−7.17E−03
Surface width (ft)	−0.01258	7.16E−04	−0.01371	−0.01139
Posted speed limit (miles/h)	0.03116	5.25E−03	0.02238	0.03970
Posted speed limit squared (miles^2/h^2)	*−1.40E−05*	*1.57E−05*	*−4.02E−05*	*1.19E−05*
Average annual daily traffic (AADT)	1.08E−04	3.28E−06	1.03E−04	1.13E−04
Indicator for minor arterial: 1 = yes, 0 = otherwise	0.2257	0.02809	0.1799	0.2729
Indicator for collector: 1 = yes, 0 = otherwise	0.4971	0.03114	0.4448	0.5478
Indicator for rolling terrain: 1 = yes, 0 = otherwise	−0.2344	0.02530	−0.2756	−0.1934
Indicator for mountainous terrain: 1 = yes, 0 = otherwise	−0.3552	0.1301	−0.5677	−0.1452
Number of observations			7773	

Note: Italics are used for parameters that do not differ from zero in a statistically significant way, based on the 95% (2.5–97.5) sample-based credible sets.

Table 4
Non-disabling injury crash frequency MVPLN model results

Variable definition	Mean	Standard error	The 95% (2.5–97.5%) sample-based credible sets	
Constant	−15.37	0.9321	−16.89	−13.81
Horizontal curve length (ft)	−2.01E−05	2.41E−06	−2.41E−05	−1.61E−05
Degree of curvature (°/100 ft)	0.1576	6.04E−03	0.1477	0.1676
Vertical curve length (ft)	−2.04E−04	1.12E−05	−2.22E−04	−1.85E−04
Vertical grade (%)	0.1850	0.01532	0.1602	0.2110
Average shoulder width (ft)	−4.69E−03	9.17E−04	−6.22E−03	−3.22E−03
Surface width (ft)	−0.01079	1.25E−03	−0.01287	−8.72E−03
Posted speed limit (miles/h)	0.01335	1.73E−03	0.01051	0.01621
Posted speed limit squared (miles^2/h^2)	*−2.30E−04*	*1.56E−04*	*−4.82E−04*	*3.38E−05*
Average annual daily traffic (AADT)	*2.37E−06*	*3.55E−06*	*−3.46E−06*	*8.24E−06*
Indicator for minor arterial: 1 = yes, 0 = otherwise	0.2489	0.02867	0.2025	0.2963
Indicator for collector: 1 = yes, 0 = otherwise	0.4896	0.03679	0.4292	0.5508
Indicator for rolling terrain: 1 = yes, 0 = otherwise	0.1341	0.02343	0.09553	0.1733
Indicator for mountainous terrain: 1 = yes, 0 = otherwise	*−0.1685*	*0.1100*	*−0.3428*	*0.01523*
Number of observations			7773	

Note: Italics are used for parameters that do not differ from zero in a statistically significant way, based on the 95% (2.5–97.5) sample-based credible sets.

(an open-source statistical computing environment described at http://www.r-project.org/)[10]. The prior distributions for the estimation are defined by the hyperparameters $\nu_\Sigma = 10$, $V_\Sigma^{-1} = I_5$, $\beta_{0s} = (0, 0, \ldots, 0)'$, and $V_{\beta_{0s}} = 100 \times I_{14}$. The Gibbs sampler was implemented to obtain $M = 8000$ draws for Σ. The two M–H algorithms were implemented to obtain $M = 8000$ draws for each of the $5 \times 14 = 70$ β's and each of the $7773 \times 5 = 38{,}865$ ε's, respectively. The initial 1000 draws were discarded as "burn-ins." An adequate burn-in period eliminates the influence of the starting values. To help ensure chain convergence, the Gibbs sampler and the two M–H algorithms were implemented using two sets of starting values[11] and both converged at the same posterior distribution of parameters. Convergence was obtained fairly quickly, under a variety of assumed priors and initial values. And standard diagnostic methods, as proposed by various researchers (including Gelman et al., 2004), were used, such as checking for autocorrelations and cross-correlations. Estimation results are presented in Tables 2–6.

Based on the posterior density of Σ, positive correlations between crash counts at different levels of severity within the segment do appear to exist, in a statistically significant way. The univariate models are a special case of the MVPLN, with off-diagonal elements of Σ equal to zero. Given the MVPLN predictions' added flexibility to represent such pattern, it is expected that they offer somewhat better predictions.

5.2. Interpretation of results

The following discussion of results emphasizes disabling and fatal injuries (Tables 5 and 6), since these arguably are of greatest concern to agencies and policymakers. Moreover, the data on such outcomes are more likely to be reported and more reliably recorded than that for other crash outcomes (Blincoe et al., 2002). Tables 2–4 provide crash count model estimates for the

[10] The source code can be found in Ma (2006).
[11] Zeros were used as the starting values for β in the second chain.

Table 5
Disabling injury crash frequency MVPLN model results

Variable definition	Mean	Standard error	The 95% (2.5–97.5%) sample-based credible sets	
Constant	−16.73	2.182	−20.37	−13.12
Horizontal curve length (ft)	6.49E−05	3.97E−05	3.70E−07	1.30E−04
Degree of curvature (°/100 ft)	0.02029	6.64E-03	9.62E-03	0.03097
Vertical curve length (ft)	−3.69E−04	3.63E−05	−4.28E−04	−3.10E−04
Vertical grade (%)	0.1431	0.01101	0.1255	0.1607
Average shoulder width (ft)	*6.27E−03*	*0.01656*	*−0.02102*	*0.03334*
Surface width (ft)	−9.85E−03	1.47E−03	−0.01226	−7.41E−03
Posted speed limit (miles/h)	0.01040	1.81E−03	7.42E−03	0.01344
Posted speed limit squared (miles^2/h^2)	*3.48E−04*	*3.22E−04*	*−1.94E−04*	*8.64E−04*
Average annual daily traffic (AADT)	5.34E−04	5.78E−05	4.38E−04	6.30E−04
Indicator for minor arterial: 1 = yes, 0 = otherwise	0.3470	0.04676	0.2700	0.4243
Indicator for collector: 1 = yes, 0 = otherwise	0.4106	0.05675	0.3171	0.5033
Indicator for rolling terrain: 1 = yes, 0 = otherwise	0.2814	0.04212	0.2133	0.3498
Indicator for mountainous terrain: 1 = yes, 0 = otherwise	*167.6*	*115.3*	*−24.93*	*355.2*
Number of observations			7773	

Note: Italics are used for parameters that do not differ from zero in a statistically significant way, based on the 95% (2.5–97.5) sample-based credible sets.

other three severity levels. The signs of most coefficients are consistent throughout the models, indicating robust directions of effect for most control variables.

Parameter estimates shown in Tables 2–6 suggest that roadway design plays an important role in predicting crash counts. For example, holding all other factors fixed, more severe injury crashes are expected on sharper horizontal curves, while wider shoulders tend to reduce rates of less severe crashes (perhaps by offering added maneuverability space for crash avoidance). Based on an average road segment's attributes and the MVPLN model's average parameter estimates, Table 7 provides estimates of percentage changes in crash rates as a function of various design details. For example, a 5-ft increase in (average) right shoulder width (from 2 to 7 ft) is predicted to result in 7.04% fewer crashes (total) per 100 million VMT. A 26.6% higher average annual daily traffic level (rising from 3757 to 4757 vehicles) is predicted to increase total crash count by 16.4%—while reducing the total crash *rate* by 5.51%. In this way, the MVPLN model results offer statistically (and practically) significant insights into crash counts' dependence on roadway design.

The magnitudes of the parameter estimates for the MVPLN specification are not directly comparable to those of univariate Poisson models (shown in Ma, 2006) or those of univariate negative binomial (UVNB) models (also shown in Ma, 2006). The reason for this is that the MVPLN model accounts for correlations across crash counts (by severity), and is therefore somewhat different from the univariate cases. However, a comparison of parameter signs shows that sharper curves are associated with more fatal crashes in all three models (MVPLN, UVP, and UVNB). The rest of control variables are not statistically significant in both the UVP and UVNB models; however, some of these control variables remain showing a statistically significant effect on fatal crash occurrence in the MVPLN model. For example, speed limit is not statistically significant in the univariate models but is expected to increase fatal crash rates in the MVPLN model. Vertical curve length and segment grade show the same pattern of effects on disabling-injury crashes in all

Table 6
Fatal crash frequency MVPLN model results

Variable definition	Mean	Standard error	The 95% (2.5–97.5%) sample-based credible sets	
Constant	−24.46	6.780	−35.61	−13.63
Horizontal curve length (ft)	−3.56E−05	5.67E−06	−4.47E−05	−2.63E−05
Degree of curvature (°/100ft)	0.02080	1.23E−03	0.01868	0.02274
Vertical curve length (ft)	3.67E−05	1.07E−05	1.93E−05	5.39E−05
Vertical grade (%)	−0.05849	0.02737	−0.1032	−0.01380
Average shoulder width (ft)	*0.01766*	*0.03147*	*−0.03503*	*0.06981*
Surface width (ft)	0.05338	0.02102	0.01937	0.08909
Posted speed limit (miles/h)	0.01463	2.27E−03	0.01073	0.01835
Posted speed limit squared (miles^2/h^2)	*1.78E−04*	*9.08E−04*	*−1.34E−03*	*1.64E−03*
Average annual daily traffic (AADT)	*1.64E−05*	*1.30E−05*	*−4.62E−06*	*3.83E−05*
Indicator for minor arterial: 1 = yes, 0 = otherwise	0.1532	0.09024	3.70E-03	0.3053
Indicator for collector: 1 = yes, 0 = otherwise	0.4176	0.1206	0.2263	0.6169
Indicator for rolling terrain: 1 = yes, 0 = otherwise	−0.1714	0.07712	−0.2997	−0.04648
Indicator for mountainous terrain: 1 = yes, 0 = otherwise	1.801	0.2251	1.436	2.172
Number of observations			7773	

Note: Italics are used for parameters that do not differ from zero in a statistically significant way, based on the 95% (2.5–97.5) sample-based credible sets.

Table 7
Expected percentage changes in crash rates corresponding to changes in variables

Variables	Averages	Changes in variable	Percentage change in crash rates (per 100 million VMT)					
			Fatal (%)	Disabling (%)	Non-disabling (%)	Possible (%)	PDO (%)	Total (%)
CURV_LGT	248 (ft)	+100	−0.36	0.65	−0.20	–	–	0.30
DEG_CURV	2.3 (°/100 ft)	+2	4.08	3.98	27.04	18.63	21.98	18.58
VCUR_LGT	303 (ft)	+100	0.37	−3.76	−2.06	−3.01	−2.08	−2.52
PCT_GRAD	1.805	+2	−12.41	24.88	30.93	27.62	24.07	24.86
SHLDWID	2.1 (ft)	+5	–	–	−5.54	−6.49	−7.89	−7.04
SURF_WID	24 (ft)	+5	−12.52	−58.65	−5.36	−6.49	4.76	0.04
SPD_LIMT	50 (mile/h)	+10	28.97	38.56	−12.72	25.64	−1.95	12.99
AADT	3757	+1000	–	41.37	–	10.24	4.68	16.42

Table 8
Correlation-coefficients of $\vec{\varepsilon}_i$

	Fatal	Disabling	Non-dsabling	Possible injury	PDO
Fatal	1	0.04207	0.01777	0.02191	0.02718
Disabling		1	0.05061	0.06100	0.4328
Non-dsabling			1	0.08071	0.1304
Possible injury				1	0.3552
PDO					1

three models. For example, long vertical curves are predicted to reduce disabling-injury crashes, but steeper segments are associated more disabling-injury crashes. The coefficient signs for remaining control variables are not in agreement across all three models, indicating that specification choice is important to a proper understanding of crash count relationships.

Based on the description of the correlation effects earlier in the paper, we should expect the MVPLN specification to yield a superior crash prediction model because the crash counts by severity on the same segment of roadway are found to be correlated with one another as shown in Table 8. Note that this is not a theoretical point, but rather an empirical one: in other words, where potential correlation exists, it should be modeled. Like the MVNB approach, our approach allows for overdispersion. The correlations may be caused by omitted variables (such as pavement quality, sight distance, driveway density, and surrounding land use), which can influence crash occurrence at all levels of severity. Essentially, higher crash rates of one type are associated with higher crash rates of other types. Negative correlations are not likely in models of crash prediction since crash likelihood for all crash types is likely to rise due to the same deficiencies in roadway design, or other unobserved factors.

Table 9
Comparisons of crash predictions from univariate and multivariate models

Observed	PDO 981	Possible 331	Non-disabling 287	Disabling 83	Fatal 23
UVP					
Prediction	1050	432.6	384.3	120.8	30.44
Difference	69.24	101.6	97.32	37.77	7.444
Percentage difference (%)	7.06	30.70	33.91	45.51	32.37
UVNB					
Prediction	1039	396.5	345.4	104.8	29.91
Difference	58	65.5	58.4	21.8	6.91
Percentage difference (%)	5.91	19.79	20.35	26.27	30.04
MVPLN1[a]					
Prediction	1013	358.2	310.1	96.8	27.13
Difference	32	27.2	23.1	13.8	4.13
Percentage difference (%)	3.26	8.22	8.05	16.63	17.96
MVPLN2[b]					
Prediction	1005	348.3	306.4	97.17	26.52
Difference	24	17.3	19.4	14.17	3.52
Percentage difference (%)	2.45	5.23	6.76	17.07	15.30

Note: A total of 13,050 rural two-lane road segments in the Puget Sound region were used for model prediction.

[a] The MVPLN1 predictions were computed as follows: (1) 1000 samples of all severity-specific parameters were taken from a multivariate normal distribution with the posterior distribution's mean and correlation correlations; (2) 1000 samples of nuisance parameters (error terms) were drawn from a multivariate normal with zero and correlation coefficients shown in Table 8; (3) expected crash counts for each segment were calculated, for all 1000 samples.

[b] The MVPLN2 predictions were obtained as follows: (1) 7000 samples of nuisance parameters (error terms) were drawn from a multivariate normal with zero mean and correlation coefficients shown in Table 8; (2) 7000 expected crash counts were computed for all segments using these 7000 draws along with the 7000 draws from the MCMC simulation.

In addition, out-of-sample predictions from both univariate and multivariate models are compared for the different groups. Table 9 suggests that the MVPLN model with MCMC draws predicts better than the univariate models (UVP and UVNB). This is because the MVPLN model addresses the issue of unobserved heterogeneity and allows for correlations among crash counts at all levels of severity.

6. Conclusions

Roadway safety is a major concern for the general public—and its transport agencies. Roadway crashes claim many lives and cause substantial economic losses each year. The situation is of particular interest on rural two-lane roadways, which experience significantly higher fatality rates than urban roads. There have been numerous efforts devoted to investigating crash occurrence as related to roadway design features, environmental conditions and traffic levels. However, almost all such research has relied on univariate count models; that is, traffic crash counts at different levels of severity have been estimated separately. The widely used univariate count data models neglect the interdependence of crash counts at different levels of severity for a specific segment of roadway.

This research simultaneously models correlated crash counts at different levels of severity using an MVPLN regression specification, which allows for a rather general correlation structure as well as overdispersion. With recent advances in crash modeling and Bayesian statistics, parameter estimation is achieved within the Bayesian paradigm, using a Gibbs Sampler and Metropolis–Hastings algorithms.

Crash counts for over 7773 homogeneous segments of rural two-lane Washington State roadways in the Puget Sound region in 2002 were used to estimate the model. Thanks to MCMC simulation techniques, the marginal posterior distributions of all parameters of interest were obtained, and estimation results from the MVPLN approach offered better predictions than those from univariate Poisson and negative binomial models.

As anticipated, the results lend themselves to several recommendations for highway safety treatments and design policies. For example, adding shoulder width is predicted to be highly cost-effective, in terms of the crash cost reductions over the long run.

The current MVPLN specification assumes no spatial correlation across roadway segments. Various unobserved variables may play very similar roles in determining crash frequency on adjacent roadway segments. The assumption of no spatial correlation is actually too strong in this case. These uncontrolled (or simply unobserved) factors may also render significant spatial correlations over time (see, e.g., Meliker et al., 2004; Miaou et al., 2003; Pawlovich et al., 1998.) Additionally, the high level of correlation between PDO and disabling crashes may indicate some ambiguity or weakness in severity classification schemes, if one believes that unobserved heterogeneity in omitted variables should generate significant correlation (e.g., in data sets with relatively few control variables available).

The framework of this research is established in its parametric assumptions. Parametric methods can be implemented using assumptions of underlying distributions and relationships. Misspecification of the distribution may lead to serious errors in subsequent data analysis. Semi-parametric and nonparametric regression analysis relaxes these assumptions[12] (see, e.g., Gurmu et al., 1999; Wooldridge, 1999; Alfò and Trovato, 2004). For example, Gurmu et al. (1999) developed a semiparametric approach to investigate overdispersed count data using a Laguerre series expansion of an unknown density function for unobserved heterogeneity.

The cost of relaxing such assumption requires more computation and, in some instances, a more difficult-to-understand result. The benefits of nonparametric methods include a potentially more accurate estimate of the regression function and often "exact" probability statements, regardless of the shape of the population distribution from which the random sample was drawn (Damien, 2005).

The MVPLN model estimated here incorporates the safety effects of several roadway design and traffic features of interest to traffic and transportation engineers. However, several features of interest that are not available have been omitted from the model, including, for example, driveway density and sight distance. In addition, the model generally treats the effects of individual geometric design features as independent of one another and ignores potential interactions among them. Such interactions may exist (such as combinations of horizontal and vertical curvature on the same segment), and these should be examined in the future endeavors of this type.

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