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IFAC-PapersOnLine 48-11 (2015) 354–359

Echo State Neural Network Based State Feedback Control for SISO Affine Nonlinear Systems

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Abstract: Echo state network (ESTN) is a new recurrent neural networks (RNN) with a simpler training method. Based on ESTN, this paper address a state feedback control algorithm for a class of perturbed SISO nonlinear systems in the affine form. The control algorithm is implemented without *a prior* knowledge of the nonlinear system. The network weights can be tuned on line by the Recursive Least Squares (RLS) method without off line learning phase needed. The convergence and the Bounded Input Bounded Output (BIBO) stability of the ESTN controller are proven. Moreover, all signals involved in the closed loop are proven to be exponentially bounded and then the stability of the system. We have used the tracking problem of one-link rigid robotic manipulator system as an example to verify the effectiveness of the proposed method.

Keywords: Echo state neural network, Direct adaptive control, State feedback control, Nonlinear systems

1. INTRODUCTION

In the past several years, neural network based adaptive control schemes for nonlinear systems have been active research area. Under the assumption that all states of the plant are available for measurement, adaptive neural network state feedback control schemes have been developed as in Ge et al. (1999),Wai (2003), Tang et al. (2006), Belarbi and Chemachema (2007), and Wang et al. (2014). When only the system output can be measured, the observer-based adaptive neural network control were proposed for a certain class of unknown nonlinear systems for example Sridhar and Khalil (2000), Chien et al. (2011),Castaneda and Esquivel (2012), and Chemachema (2012). In the case of multiple input-multiple output nonlinear systems, many adaptive output feedback control schemes were proposed as in Chen et al. (2010), Li et al. (2011) and Kostarigka and Rovithakis (2012).

The multilayer perceptron (MLP) and the RBF neural networks are the most popular topologies used in these schemes, but those available have a static feed forward network structure that requires a large number of neurons. Furthermore, the weight updates of them do not use the internal neural netwok information. To address these drawbacks, recurrent NNs (RNNs) Williams and Peng (1990), Ku and Lee (1995), and Lee and Teng (2000) have been introduced to apply dynamic feedback structures in which the present activation state is a function of the previous activation state and of the present inputs. The recurrent neural networks have been developed in many adaptive control schemes Chin-Min et al. (2012), Li et al. (2014), Chun-Fei (2014), and Michael et al. (2014).

Recently, the echo state network (ESTN) proposed in Jaeger (2001), is an advanced recurrent neural network algorithm. The ESTN is used as dynamic reservoirs with a large number of neurons that are randomly interconnected and/or self connected. The reservoir itself is fixed, once it is selected. During the training process of ESTN, only the output connections are trained through on line linear regression or on line methods, such as the recursive least square (RLS). Unlike other recurrent architectures, the use of random weights in the ESTN and the training algorithm greatly simplify the training of the network and eliminate the issues of stability, convergence, and local minima. The ESNT has been successfully applied for many applications. The ESTN has been developed in predication the chaotic time series Jaeger and Haas (2004), and Zhiwei and Min (2007). In the identification and control of dynamic systems, it has been used as in Jaeger et al. (2007), Xu et al. (2005), Venayagamoorthy (2007), Pan and Wang (2012), Seong and Lee (2013), and Seong and Lee (2014).

This paper addresses the development of the ESTN in the state feedback control scheme for a special class of SISO nonlinear systems. We develop the ESTN with on line updating law to approximate an ideal controller deduced from the certainly equivalent approach. Likewise to the method proposed in Belarbi and Chemachema (2007) and Chemachema (2012), we propose a simple method to

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2015.09.211

estimate the control error to derive the updating laws of the neural network controller. Using the estimated control error, the recursive least squares method is used to train the output weights of the ESTN controller. According to the main features of the ESTN, the convergence of the output weights of the controller is easy to prove. Furthermore, the Lyapunov direct method is then used to prove the global exponential boundedness of all the signals involved in the closed loop, and hence the stability of the system. The performance of the proposed scheme is evaluated using one-link rigid robotic manipulator system. The paper is organized as follows. Section (2) describes the structure and the learning of the ESTN. In section (3), we describe the nonlinear system and the problem formulation. The proposed state feedback control based on the ESTN is presented in section (4). The simulation results of the proposed scheme is introduced in section (5). Section (6) provides a summary of the proposed work.

2. THE ECHO STATE NEURAL NETWORK

This section describes the structure and the learning of the ESTN.

2.1 ESTN Structure

The ESTN composes of a hidden layer (dynamical reservoir) with randomly interconnected neurons and a memoryless output layer (readout). Let K, N, L represent the number of input, reservoir, and output units, respectively, $z(t) = [z_1(t), ..., z_K(t)]^T$ are the K-dimensional external input, $v(t) = [v_1(t), ..., v_N(t)]^T$ are the N-dimensional reservoir activation state, and $y(t) = [y_1(t), ..., y_L(t)]^T$ are the L-dimensional output vector at time step t. The state and output equations of the ESTN are

$$v(t+1) = f(\mathbf{W}v(t) + \mathbf{W}^{in}z(t) + \mathbf{W}^{bias}) \tag{1}$$

$$y(t) = \mathbf{W}^{out}v(t+1) \tag{2}$$

where $f(.)$ is the activation function (typically tanch functions), $\mathbf{W} \in \Re^{N\times N}$, $\mathbf{W}^{in} \in \Re^{N\times K}$, $\mathbf{W}^{bias} \in \Re^{N\times 1}$ and $\mathbf{W}^{out} \in \Re^{L\times N}$ denote internal connection, input connection, bias, and the output connection weights, respectively. An additional nonlinearity can be applied to $y(t)$ in (2), as well as feedback connection from output $y(t-1)$ to the internal state $v(t)$ in (1).

Definition 1. For an ESTN in (1) trained by using many set of input vector $z(t)$ in compact intervals $\mathcal{Z}$, it has echo state property with respect to $\mathcal{Z}$ if for any infinite sequence $z(t)$, and for all state sequences $\acute{v}(t)$ and $v^{''}(t)$ associated with the reference sequence; i.e.,

$$v^{'}(t+1) = f(\mathbf{W}v^{'}(t) + \mathbf{W}^{in}z(t) + \mathbf{W}^{bias}) \tag{3}$$

$$v^{''}(t+1) = f(\mathbf{W}v^{''}(t) + \mathbf{W}^{in}z(t) + \mathbf{W}^{bias}) \tag{4}$$

the following equality holds for a while:

$$v^{'}(t) = v^{''}(t) \tag{5}$$

This definition states that, if the ESTN starts from two initial states $\acute{v}(0)$ and $v^{''}(0)$ with the same input sequence, after running for long time, the state sequence of the ESTN would converge.

2.2 ESTN Learning

The basic idea of the ESTN's learning is that the values of $\mathbf{W}$, $\mathbf{W}^{in}$, and $\mathbf{W}^{bias}$ are fixed without adaptation. Whereas, only the readout weights, $\mathbf{W}^{out}$, will be adapted. All wights matrices are randomly initialized according to the standard distribution $\sim N(0,1)$. Let λ_{max}, the largest absolute eigenvalue (called the spectral radius) of $\mathbf{W}$. If $|\lambda_{max}| > 1$, then the network (1) has no echo state property for any $\mathcal{Z}$. Experimental results deduced that if $|\lambda_{max}| < 1$ is a sufficient condition for the echo state property Jaeger (2001). Thus, after initialization, the matrix $\mathbf{W}$ is normalized by dividing it with λ_{max}.
The learning of the output weights $\mathbf{W}^{out}$ in (1) can be expressed as solving a system of linear equations by

$$Y_{target} = \mathbf{W}^{out}V \tag{6}$$

with respect to $\mathbf{W}^{out}$, where $V \in \Re^{N\times T}$ is all $v(t)$ produced by presenting the reservoir with $z(t)$ in (1), and $Y_{target} \in \Re^{L\times T}$ are all $Y_{target}(t)$, both collected into respective matrices over the training period $t = 1, ..., T$. Equation.(6) can be solved using a method for finding least squares solutions of the optimization problem

$$min_{\hat{W}^{out}} = \left\|\mathbf{W}^{out}V - Y_{target}\right\| \tag{7}$$

3. PROBLEM FORMULATION

Consider the n^{th} order nonlinear dynamical system expressed in the affine form

$$\begin{aligned} \dot{x_i} &= x_{i+1}, i = 1, 2, ..., n-1 \\ \dot{x_n} &= f(x) + g(x)u + d \\ y &= x_1 \end{aligned} \tag{8}$$

where $x = [x_1, x_2, ..., x_n] \in \Re^n$, $u \in \Re$, $y \in \Re$ are the state variables, control input, and system output, respectively. d is external bounded disturbance. Equation. (8) can be rewritten in the following state space form

$$\begin{aligned} \dot{\mathbf{x}} &= A\mathbf{x} + B(f(\mathbf{x}) + g(\mathbf{x})u + d) \\ y &= C^T\mathbf{x} \end{aligned} \tag{9}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & . & . & . & 0 \\ 0 & 0 & 1 & . & . & . & 0 \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & . & 1 \\ 0 & 0 & 0 & . & . & . & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ . \\ 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 \\ 0 \\ . \\ . \\ . \\ 0 \\ 0 \end{bmatrix}$$

In this paper, the following assumption is taken for the system in (8)

Assumption 1. $f(\mathbf{x})$ and $g(\mathbf{x})$ are unknown nonlinear and bounded function, $g(\mathbf{x}) \neq 0$ for all values of $\mathbf{x}$. The output is required to follow a bounded smooth trajectory y_r, d is bounded disturbance and the derivatives of y_r up to n^{th} order (i.e.,$y_r^{(n)}$) exist and bounded.

Define the desired output and the tracking error vector as

$$\begin{aligned} \mathbf{y}_r &= [y_r, \dot{y_r}, ..., y_r^{(n-1)}] \in \Re^n \\ \mathbf{e} &= \mathbf{y}_r - \mathbf{x} \\ e_1 &= y_r - x_1 \end{aligned}$$

Based on the certainty equivalent approach, an optimal control law is

$$u^* = \frac{1}{g(\mathbf{x})}[-f(\mathbf{x}) + y_r^{(n)} + \mathbf{K}_c^T \mathbf{e}] \quad (10)$$

where $\mathbf{K}_c$ is the state feedback gain vector, chosen that the matrix $A - B\mathbf{K}_c$ is Hurwitz. Since $f(x)$ and $g(x)$ are unknown, the control law (10) cannot be implemented. For the system output y to follow a given reference trajectory y_r effectively, we propose a direct adaptive echo state neural network controller in the following section.

4. STATE FEEDBACK CONTROLLER BASED ON ECHO STATE NEURAL NETWORK

Consider an ESTN with a single output is designed to estimate the control law in (10) such that its reservoir state and output equations are

$$v(t+1) = f(\mathbf{W}v(t) + \mathbf{W}^{in} z(t)) + \mathbf{W}^{bias}) \quad (11)$$

$$\hat{u}(t) = \mathbf{W}^{out} v(t+1) \quad (12)$$

where the input vector $z(t)$ of the ESTN controller is selected as $[-\mathbf{x}^T, y_r^{(n)}, \mathbf{K}_c^T \mathbf{e}]^T$, $\mathbf{x}^T = [x_1, x_2, ..., x_n]$ is the measured state vector, and $\mathbf{K}_c$ is the state feedback gain vector, $\mathbf{e} = \mathbf{y}_r - \mathbf{x}$, and $f(.)$ is the symmetric *tanh* activation function.

4.1 Weight Update

Here, we present the adaptation law of the output weights $\mathbf{W}^{out}$ of the ESTN controller in the proposed scheme. Define the control error $\tilde{u}$ between the optimum control law in Eq. (10) and the estimated control law in Eq.(12) associated with the output weights $\mathbf{W}^{out}$

$$\tilde{u}(t) = u^*(t) - \hat{u}(t) \quad (13)$$

With each iteration, we use the RLS algorithm to update the output weights of the ESTN controller as

$$\mathbf{P}(0) = \frac{\mathbf{I}}{\alpha} \quad (14)$$

$$\mathbf{W}^{out}(t) = \mathbf{W}^{out}(t-1) + \tilde{u}(t)\mathbf{P}(t)v(t) \quad (15)$$

$$\mathbf{P}(t) = \frac{\mathbf{P}(t-1)}{\zeta}\left(1 - \frac{v(t)v^T(t)\mathbf{P}(t-1)}{\zeta + v^T(t)\mathbf{P}(t-1)v(t)}\right) \quad (16)$$

where $v(t)$ is the current reservoir state, ζ is the forgetting factor, and α is an initially chosen value. $\mathbf{P}(t)$ is the covariance matrix that is the running estimate of the Moore-Penroose pseudo-inverse of $(vv^T + \eta\mathbf{I})$ with η a regularization parameter Waegeman et al. (2012) and $\mathbf{P}(0)$ denotes the initial value of $\mathbf{P}(t)$.

4.2 Control Error Estimator

Due to the ideal controller defined in Eq. (13) can not be computed, an estimate $\tilde{u}_{est}(t)$ can be used instead in the adaptive law (15). In Belarbi and Chemachema (2007) and Chemachema (2012), the authors proposed a fuzzy inference method with heuristically rules to obtain an estimate value of the ideal control error. Since, the tracking error and its rate of change are used as the inputs of this fuzzy inference method. We propose here a simple method to estimate the ideal control error that its computational cost would be lower than the fuzzy logic method in Belarbi and Chemachema (2007) and Chemachema (2012). This method supposes that the value $\tilde{u}_{est}(t)$ can be estimated directly as

$$\tilde{u}_{est}(t) = g_1 e_1 + g_2 \dot{e_1} \quad (17)$$

where $e_1 = y_r - y$, g_1 and g_2 are small real values. The sign of both g_1 and g_2 will be selected positive in case of positive control gain systems while it will be negative for negative control gain systems. According to both $\tilde{u}(t)$ and $\tilde{u}_{est}(t)$ are scalers and (17) guarantees the latter has the correct sign of $\tilde{u}(t)$, we can write

$$\tilde{u}(t) = S\tilde{u}_{est}(t) \quad (18)$$

where S is a positive scaling factor. Selecting this scaling factor will only affect the step size of the updating law.

4.3 Convergence and Stability Analysis

In this sub-section, we will investigate the bounded input bounded output (BIBO) stability and the convergence of the ESNT controller. The stability conditions of the closed loop system also will be given latter.

In order to discuss the BIBO stability of the ESTN controller, consider the reservoir state has the following linear form

$$v(t+1) = \mathbf{W}v(t) + \mathbf{W}^{in} z(t) \quad (19)$$

As stated befor, λ_{max} is defined as the largest absolute eigenvalue (called the spectral radius) of $\mathbf{W}$. According to the linear system theory, if $|\lambda_{max}| < 1$, the linear system (19) will be bounded Zhiwei and Min (2007). For different eigen values of $\mathbf{W}$, the linear system (19) has different transient properties. For the reservoir (11) with the *tanh* activation function, we have

$$\|tanh(\mathbf{W}v(t))\| \leq \|\mathbf{W}v(t)\| \quad (20)$$

Hence, due the nonlinearity in Eq.(11) (i.e., *tanh* activation function) and the condition the spectral radius of $\mathbf{W}$ is smaller than 1, we can conclude that the ESTN output is bounded for all inputs.

The convergence of the network error defined in Eq. (13) can be discussed as follows. Using Eq.(12) and Eq.(13) we can write

$$\mathbf{W}^{out}(t-1) = \frac{u^*(t) - \tilde{u}(t)}{v(t)} \quad (21)$$

Substituting (21) in (15), the error after the weight update is

$$\mathbf{W}^{out}(t) = \frac{u^*(t) - \tilde{u}(t)}{v(t)} + \tilde{u}(t)\mathbf{P}(t)v(t) \quad (22)$$

$$\mathbf{W}^{out}(t)v(t) = u^*(t) - \tilde{u}(t) + \tilde{u}(t)v(t)^T\mathbf{P}(t)v(t) \quad (23)$$

$$\tilde{u}(t) = \frac{u^*(t) - \mathbf{W}^{out}(t)v(t)}{(1 - v(t)^T\mathbf{P}(t)v(t))} \quad (24)$$

$$\tilde{u}(t) = \frac{\tilde{u}_+(t)}{(1 - v(t)^T\mathbf{P}(t)v(t))} \quad (25)$$

$$\tilde{u}_+(t) = \tilde{u}(t)(1 - v(t)^T\mathbf{P}(t)v(t)) \quad (26)$$

We have mentioned that $\mathbf{P}(t)$ is running estimate of the Moore-Penroose pseudo-inverse of $(vv^T + \eta\mathbf{I})$, then we can write

$$\mathbf{P}(t) = \left(v(t)v(t)^T + \eta\mathbf{I}\right)^{-1} \quad (27)$$

Due to using the tanh activation function in (11), we know that $|v(t)| \leq 1$. As a result $v(t)^T\mathbf{P}(t)v(t)$ in (27) will change from a value close to 1 to a value that asymptotically converge to 0. Consequently, $\tilde{u}_+(t)$ will become small and will converge to local minimum (i.e., eventually converge to $\tilde{u}(t)$). Thus the change of the output weight $\mathbf{W}^{out}(t) - \mathbf{W}^{out}(t-1)$ becomes 0.

In the following, we will discuss the convergence of the tracking error for the closed loop system. The following theorem summarizes the stability results of the closed loop system.

Theorem 4.1. For a nonlinear SISO system (8), the ESTN controller (12) and the adaptive law (15), if the initial values of all ESTN controller weights (W^{out}, W, and W^{in}) are bounded and the tanh activation function are used in (11), then all the signals in the closed loop system are bounded and the tracking error converges exponentially to a bounded compact as $t \to \infty$.

Proof. Substituting $\hat{u}$ and u^* in system (9) yields the following error dynamics

$$\begin{aligned} \dot{\mathbf{e}} &= A_c\mathbf{e} + B(g(\mathbf{x})(u^* - \hat{u}) - d) \\ \mathbf{e}_1 &= C^T\mathbf{e} \end{aligned} \tag{28}$$

where $A_c = A - B\mathbf{K}_c^T$. Using the control error defined in (13), the above equation can be rewritten as:

$$\begin{aligned} \dot{\mathbf{e}} &= A_c\mathbf{e} + B(g(\mathbf{x})\tilde{u} - d) \\ \mathbf{e}_1 &= C^T\mathbf{e} \end{aligned} \tag{29}$$

Define the following Lyapunov function

$$V_e = \frac{1}{2}\mathbf{e}^T\mathbf{P}_e\mathbf{e} \tag{30}$$

where $\mathbf{P}_e$ is the solution of the Lyapunov equation

$$A_c^T\mathbf{P}_e + \mathbf{P}_eA_c = -\mathbf{Q} \tag{31}$$

$\mathbf{Q}$ is an arbitrary positive definite matrix. Differentiating V_e with respect to time along the tracking error (28) gives

$$\begin{aligned} \dot{V}_e = \frac{1}{2}\dot{\mathbf{e}}^T\mathbf{P}_e\mathbf{e} + \frac{1}{2}\mathbf{e}^T\mathbf{P}_e\dot{\mathbf{e}} &= -\frac{1}{2}\mathbf{e}^T\mathbf{Q}\mathbf{e} + \mathbf{e}^T\mathbf{P}_eB(g(\mathbf{x})\tilde{u} - d) \\ &\leq -\frac{1}{2}\mathbf{e}^T\mathbf{Q}\mathbf{e} + \left|\mathbf{e}^T\right| \left|\mathbf{P}_eB\right| \left(\left|g(\mathbf{x})\tilde{u}\right| + |d|\right) \end{aligned} \tag{32}$$

Recall the following inequality of the positive definite matrix $\mathbf{Q}$ and the vector $\mathbf{e}$

$$\lambda_{min}(\mathbf{Q})\left|\mathbf{e}^2\right| \leq \mathbf{e}^T\mathbf{Q}\mathbf{e} \leq \lambda_{max}(\mathbf{Q})\left|\mathbf{e}^2\right|$$

Then we can write (34) as

$$\begin{aligned} \dot{V}_e &\leq -\frac{1}{2}\lambda_{min}(\mathbf{Q})\left|\mathbf{e}^2\right| + \left|\mathbf{e}^T\right| \left|\mathbf{P}_eB\right| \left(\left|g(\mathbf{x})\tilde{u}\right| + |d|\right) \\ &\leq -\frac{1}{2}\frac{\lambda_{min}(\mathbf{Q})}{\lambda_{max}(\mathbf{P}_e)}\lambda_{max}(\mathbf{P}_e)\left|\mathbf{e}^2\right| + \left|\mathbf{e}^T\right| \left|\mathbf{P}_eB\right| \left(\left|g(\mathbf{x})\tilde{u}\right| + |d|\right) \\ &\leq -\frac{1}{2}\frac{\lambda_{min}(\mathbf{Q})}{\lambda_{max}(\mathbf{P}_e)}\mathbf{e}^T\mathbf{P}_e\mathbf{e} + \left|\mathbf{e}^T\right| \left|\mathbf{P}_eB\right| \left(\left|g(\mathbf{x})\tilde{u}\right| + |d|\right) \\ &\leq -\frac{1}{2}\frac{\lambda_{min}(\mathbf{Q})}{\lambda_{max}(\mathbf{P}_e)}V_e + \left|\mathbf{e}^T\right| \left|\mathbf{P}_eB\right| \left(\left|g(\mathbf{x})\tilde{u}\right| + |d|\right) \end{aligned} \tag{33}$$

where $\lambda_{min}(\Pi)$ and $\lambda_{max}(\Pi)$ are the minimum and the maximum eigen values of a matrix Π, respectively.

Define $\beta_1 = \frac{\lambda_{min}(\mathbf{Q})}{2\lambda_{max}(\mathbf{P}_e)}$ and $\beta_2 = \left|\mathbf{P}_eB\right| \left(\left|g(\mathbf{x})\tilde{u}\right| + |d|\right)$, then we can rewrite (32) as

$$\dot{V}_e \leq -\beta_1V_e + \beta_2\left|\mathbf{e}^T\right| \tag{34}$$

By solving the above inequality , we find

$$V_e \leq \frac{\beta_2\left|\mathbf{e}^T\right|}{\beta_1} + \left(V_e(0) - \frac{\beta_2\left|\mathbf{e}^T\right|}{\beta_1}\right)e^{-\beta_1t} \tag{35}$$

This results that as $t \to \infty$, we find that $|V_e| \leq \frac{\beta_2\left|\mathbf{e}^T\right|}{\beta_1}$. Also we have $V_e \geq \lambda_{min}(\mathbf{P}_e)\left|(\mathbf{e}^T)\right|^2$. Then

$$\lambda_{min}(\mathbf{P}_e)\left|\mathbf{e}^T\right|^2 \leq V_e \leq \frac{\beta_2\left|\mathbf{e}^T\right|}{\beta_1}$$

As a result, we have

$$\left|\mathbf{e}^T\right| \leq \frac{1}{\lambda_{min}(\mathbf{P}_e)}\frac{\beta_2}{\beta_1} \tag{36}$$

Since $\tilde{u}$ will converge to a local minimum according to the RLS training algorithm as described above and both $g(x)$ and d are bounded, we conclude that the tracking errors converge exponentially to the bounded region given by

$$\Omega(\mathbf{e}) = \left\{\mathbf{e}/\left|\mathbf{e}^T\right| \leq \frac{1}{\lambda_{min}(\mathbf{P}_e)}\frac{\beta_2}{\beta_1}\right\} \tag{37}$$

To summarize, Fig. 1 describes the overall scheme of the proposed state feedback control scheme.

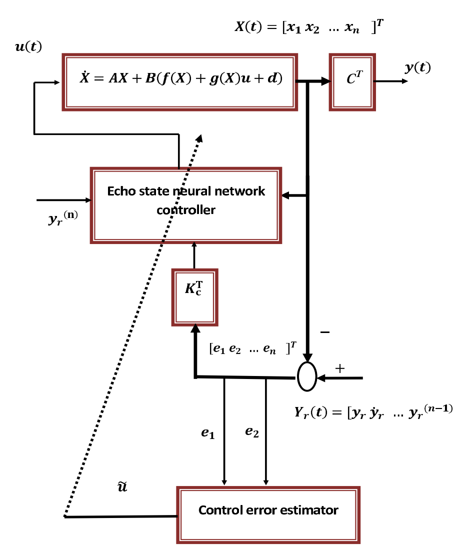

Fig. 1. The control structure of the proposed state feedback controller

5. SIMULATION RESULTS

To study the efficiency of the proposed controller, we will address the tracking problem of a robot manipulator system. This example is used in Zhihong et al. (1998), Lin and Peng (2004) , and Chemachema (2012) to demonstrate the performance of the adaptive controllers suggested therein. Few parameters have to be defined for the implementation of the proposed scheme i.e., number of reservoir of the

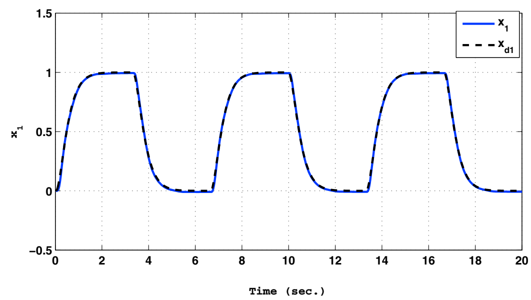

Fig. 2. The tracking of the state x_1 of the one-link rigid robotic manipulator with the proposed controller

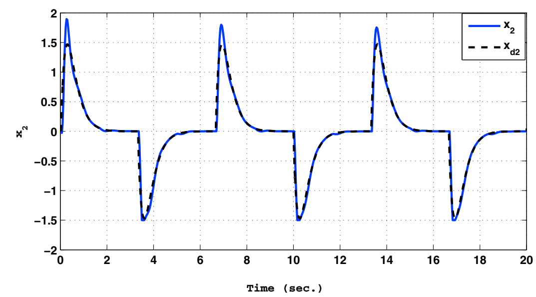

Fig. 3. The tracking of the state x_2 of the one-link rigid robotic manipulator with the proposed controller

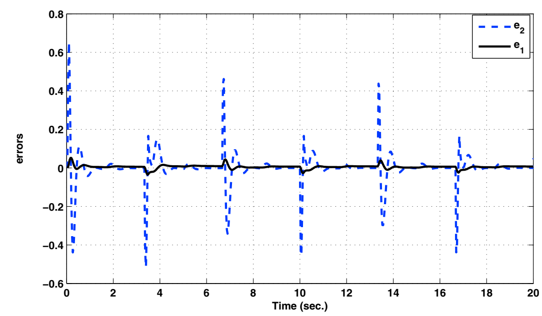

Fig. 4. The tracking errors of the one-link rigid robotic manipulator with the proposed controller

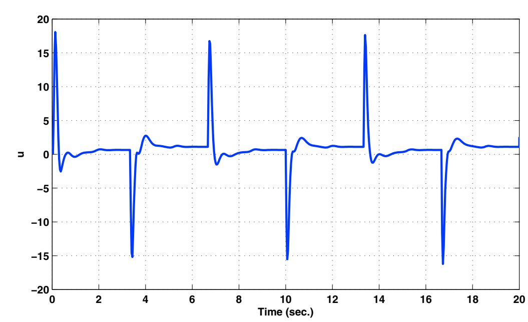

Fig. 5. The control signal of the one-link rigid robotic manipulator with the proposed controller

ESTN controller N, the state feedback gain $\mathbf{K}_c$, and the parameters of the control error estimator g_1, g_2. For choosing the number of reservoir (i.e., neurons) of the ESTN controller, a trade off between the computation complexity and the performance has to be made. Thus, we select it as $N = 50$ neurons. Also, we select $g_1 = 0.8, g_2 = 0.1$, as a small positive values to prevent the output response from having oscillation and overshoots. The fixed weights $\mathbf{W}, \mathbf{W}^{in}$ of the ESTN controller are randomly chosen according to a normal distribution $\sim N(0, 1)$. For the sake of the network convergence, after initialization, the matrix $\mathbf{W}$ is normalized by dividing it with its largest absolute eigen value. The dynamic equation of the simulation example is given by

$$ml^2\ddot{q} + b\dot{q} + mlg_v cos(q) = u \tag{38}$$

where l is the length of the link, m is the mass, and q is the angular position with initial values $q(0) = 0$ and $\dot{q}(0) = 0.1$. The above equation can be written as

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= f(x) + g(x)u(t) + d(t) \end{aligned} \tag{39}$$

where $x_1 = q, x_2 = \dot{q}$, $f(x) = (-b/ml^2)x_2 - (g_v/l)cos(x_1)$, $g(x) = 1/ml^2$, and d is the external disturbance. The system parameters are chosen as $m = l = b = g_v = 1$. The desired output of the system is given by the following reference model

$$\begin{aligned} \dot{x_{d1}} &= x_{d2} \\ \dot{x_{d2}} &= -16x_{d1} - 8x_{d2} + r(t) \end{aligned} \tag{40}$$

where $[x_{d1}(0), x_{d2}(0)]^T = [0, 0]^T$ and $r(t)$ is a periodic rectangular signal. The state feedback controller gain are chosen as $\mathbf{K}_c = [40\ \ 4]$ and the external disturbance is assumed to be a square wave with amplitude ± 0.1 with the same period of the periodic rectangular signal $r(t)$. Here, we choose the adaptive ESTN controller with the input vector $z(t) = [-x_1, -x_2, \ddot{x_{d2}}, \mathbf{K}_c^T \mathbf{e}]$. Fig. 2 and Fig. 3 illustrate the tracking of the system states x_1 and x_2, respectively. The tracking errors depicted in Fig. 4 show that the tracking errors are very small and are bounded. The obtained results shows good performance tracking despite the system disturbance. Also, the control signal shown in Fig. 5 is smooth and bounded.

6. CONCLUSION

In this paper, we address the direct adaptive control in the form of state feedback for a class of SISO perturbed nonlinear systems. The proposed design has the following features: (1) The ESTN controller used in the design scheme that is a new type of recurrent neural network, has been trained with ease and precision without changing weights between hidden and input layers. Thus, the proposed scheme is more suitable for real time implementation. (2) The ESTN controller is implemented without a priori knowledge of nonlinear systems and off line training of the network. (3) The global exponential convergence of the tracking error and the stability of the closed loop system can be easily proved using the Lyapunove method without using any supervised controller. The simulation example showed that the proposed scheme was able to perform good tracking. Furthermore, a successful control and a desired performance can be achieved in the presence of the system disturbances. In most practical situations, the all states are difficult to measure, hence a future work will develop the scheme in the output feedback form.

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