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Short Communication

Bursting pressure of mild steel cylindrical vessels

T. Aseer Brabin [a], T. Christopher [b], B. Nageswara Rao [c,*]

[a] Faculty of Mechanical Engineering, C.S.I. Institute of Technology, Thovalai 629 302, India
[b] Faculty of Mechanical Engineering, Government College of Engineering, Tirunelveli 627 007, India
[c] Structural Analysis and Testing Group, Vikram Sarabhai Space Centre, Trivandrum 695 022, India

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ABSTRACT

An accurate prediction of the burst pressure of cylindrical vessels is very important in the engineering design for the oil and gas industry. Some of the existing predictive equations are examined utilizing test data on different steel vessels. Faupel's bursting pressure formula is found to be simple and reliable in predicting the burst strength of thick and thin-walled steel cylindrical vessels.

1. Introduction

Being inexpensive and possessing high plasticity, toughness as well as good weldablity, mild steels have become the main production materials of pressure vessels such as tower reactors and exchangers or chemical equipment. The burst pressure evaluation of vessels has formed the subject of a large number of researchers to improve design precision for utilizing the maximum strength of the material.

Christopher et al. [1] examined failure data on various pressure vessels and compared the frequently used theories for validation and further use in the design of aerospace pressure vessels. Zheng and Lei [2] conducted several bursting experiments on mild steel cylindrical vessels and found inconsistency in Faupel's bursting pressure formula. Law and Bowie [3] compared several burst pressure formulae with test results of high yield-to-tensile strength ratio line pipes. Guven [4] investigated the failure pressures of thick and thin-walled copper and brass cylindrical vessels considering the Voce hardening law and plastic orthotropic effects. Zhu and Leis [5] made theoretical and numerical predictions of the burst pressure of pipes or pipelines. Since the Tresca yield theory provides a lower bound to burst pressure and the von Mises yield theory provides an upper bound, the average shear stress yield (ASSY) theory was developed for isotropic materials to improve the prediction of burst pressure. Since commercial finite element codes adopt the von Mises yield criterion and the associated flow rule as the default plasticity model for isotropic hardening metals, only the von Mises-based burst pressure of pipes can be determined using these FEA codes [6–9].

Of several formulae for calculating the burst pressure of vessels, the Faupel formula is the most popular. Based on hundreds of bursting experiments on pressure vessels made of Q235-D and 20R (1020) mild steels and after statistically analyzing the data, Zheng and Lei [2] stated that the Faupel formula had some errors. They modified the formula using the data and demonstrated its validity through comparison of test data on mild steel pressure vessels having different diameters and shell thickness. Motivated by the work of the above-mentioned researchers, this paper examines the applicability of Faupel's bursting pressure formula by considering test results of mild steel cylindrical vessels.

2. Burst pressure estimates of cylindrical pressure vessels

For power-law hardening materials, three different theoretical solutions for the burst pressure (P_b) of thin-walled pipes can be expressed in the general form [5]

$$P_b = \left(\frac{C_{ZL}}{2}\right)^{n+1} \frac{4t_i}{D_m}\sigma_{ult} \qquad (1)$$

* Corresponding author. Tel.: +91 471 2565836; fax: +91 471 2564184.
E-mail address: bnrao52@rediffmail.com (B. Nageswara Rao).

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Table 1
Comparison of failure pressure estimates with test results [3] of thin-walled end-capped steel pipes.

	X42 ex-mill	X65 aged	X70 aged	X80 ex-mill	X80 aged
Geometric details and material properties					
Outer diameter, D_o (mm)	355.65	273.14	457.20	356.90	356.17
Thickness, t_i (mm)	6.41	7.10	9.97	6.96	6.91
Ultimate tensile strength, σ_{ult} (MPa)	471	662	700	677	684
0.2% proof stress or yield strength, σ_{ys} (MPa)	321	587	637	568	640
Strain hardening exponent, n (Equation (3))	0.1415	0.0646	0.0554	0.0826	0.0445
Failure pressure, P_b (MPa) estimates and test data					
Test [3]	15.75	36.33	30.53	27.44	27.80
Tresca yield theory (Equation (1))	15.67	33.79	30.03	25.43	26.24
von Mises theory (Equation (1))	18.47	39.38	34.96	29.72	30.50
ASSY theory (Equation (1))	17.06	36.58	32.49	27.57	28.37
Svensson's formula (Equation (5))	17.82	38.58	34.29	29.02	29.93
Faupel's formula (Equation (6))	17.94	40.28	35.75	30.29	31.13
Modified Faupel's formula (Equation (9))	16.42	38.85	34.72	28.82	30.47

where t_i is the initial wall thickness; $D_m = \frac{1}{2}(D_o + D_i)$, is the mean of the inner (D_i) and outer (D_o) diameters; C_{ZL} is a yield theory-dependent constant having values

$$\begin{aligned} C_{ZL} &= 1 \quad \text{for the Tresca Theory} \\ &= \frac{2}{\sqrt{3}} \text{for the von Mises theory} \\ &= \frac{1}{2} + \frac{1}{\sqrt{3}} \text{for the average shear stress yield (ASSY) theory} \end{aligned} \tag{2}$$

σ_{ult} is the ultimate tensile strength of the material; and n is the strain-hardening exponent (usually in the range 0–0.3 for most pipeline steels) expressed in the form

$$n = 0.224\left(\frac{\sigma_{ult}}{\sigma_{ys}} - 1\right)^{0.604} \tag{3}$$

σ_{ys} is the 0.2% proof stress or yield strength of the material.

Subhananda Rao et al. [10] have obtained the burst pressure of thin-walled rocket motor cases as

$$P_b = \frac{4}{\left(\sqrt{3}\right)^{n+1}} \frac{t_i}{D_i}\sigma_{ult}, \tag{4}$$

which is same as that derived in a different way by Durban and Kubi [11] and Marin and Sharma [12]. Replacing the inner diameter (D_i) by mean diameter (D_m) in equation (4), one can obtain the failure pressure of equation (1) for the von Mises theory. Other formulae frequently used to evaluate the failure pressure of cylindrical vessels are:

Svensson [13]:

$$P_b = \sigma_{ult}\left(\frac{0.25}{n + 0.227}\right)\left(\frac{e}{n}\right)^n \ln\left(\frac{D_o}{D_i}\right) \tag{5}$$

Faupel [14]:

$$P_b = \frac{2}{\sqrt{3}}\sigma_{ys}\left(2 - \frac{\sigma_{ys}}{\sigma_{ult}}\right)\ln\left(\frac{D_o}{D_i}\right) \tag{6}$$

For relatively thin-walled vessels, a modified Svensson's formula is suggested in [8] by writing $\ln(\frac{D_o}{D_i}) \approx \frac{2t_i}{D_i}$ in equation (5). Equation (6) has been obtained using the ratio, $\frac{\sigma_{ys}}{\sigma_{ult}} : (1 - \frac{\sigma_{ys}}{\sigma_{ult}})$ to interpolate between the lowest and highest bursting pressures of the vessels (viz., $P_{\min}$ and $P_{\max}$) defined below.

$$P_{\min} = \frac{2}{\sqrt{3}}\sigma_{ys}\ln\left(\frac{D_o}{D_i}\right) \tag{7}$$

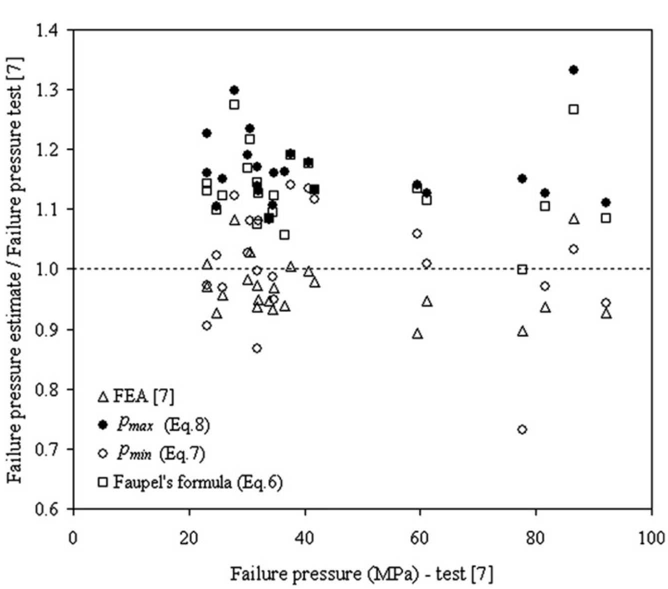

Fig. 1. Comparison of the burst pressure estimates from the Faupel's formula and FEA of Huang et al. [7] with test data.

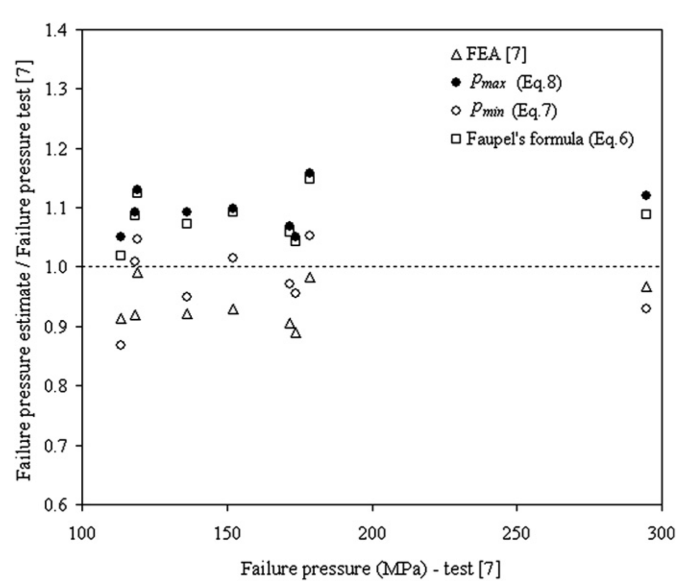

Fig. 2. Comparison of the burst pressure estimates from the Faupel's formula and FEA of Huang et al. [7] with test data.

$$P_{max} = \frac{2}{\sqrt{3}}\sigma_{ult}\ln\left(\frac{D_o}{D_i}\right) \quad (8)$$

Hill [15] suggested equation (7) for calculation of the burst strength. Aseer Brabin et al. [9] have modified Faupel's bursting pressure formula (6) in the form

$$P_b = \frac{2}{\sqrt{3}}\sigma_{ys}\left\{1+\chi\left(1-\frac{\sigma_{ys}}{\sigma_{ult}}\right)\right\}\ln\left(\frac{D_o}{D_i}\right) \quad (9)$$

where $\chi = 0.65$ for steel cylindrical vessels.

3. Results and discussion

To examine the adequacy of the bursting pressure formulae, failure data of different steel vessels are considered. Law and Bowie [3] presented failure data of thin-walled end-capped steel pipes. Table 1 gives a comparison of failure pressure estimates with test data. Tresca yield theory estimates of burst pressure are found to be close to the test results. Failure pressure estimates from the other empirical relations are found to be reasonably in good agreement with test results. The modified Faupel formula (9) predicts failure pressures close to those obtained from Svensson's formula (5).

Huang et al. [7] have compiled test data of different steels and sizes of casing to examine the adequacy of the burst pressure evaluation by performing FEA using ABAQUS. Figs. 1 and 2 shows a comparison of the burst pressure estimates from Faupel's formula (6) and FEA of Huang et al. [7] with test results. Test data are found to be within the expected P_{min} and P_{max} values from equations (7) and (8). Most of the test data are close to P_{min} values (see Table 2)

Table 2
Geometric details of casings, strength properties (yield strength, σ_{ys}; ultimate tensile strength, σ_{ult}) of different steels and comparison of failure pressure (P_{min}) estimates from equation (7) with compiled test results of burst pressure (P_b) by Huang et al. [7].

Huang et al. [7]					
Outer diameter D_o (mm)	Thickness t_i (mm)	σ_{ys} (MPa)	σ_{ult} (MPa)	P_b (MPa) Test	P_{min} (MPa) (Eq.(7))
507.93	14.30	508.8	571.0	34.50	34.09
544.05	13.50	623.9	624.0	33.84	36.67
762.40	20.00	531.5	608.0	30.63	33.07
762.40	20.00	555.0	580.0	31.95	34.54
609.60	15.90	534.3	653.0	34.79	33.05
609.60	15.90	440.5	585.0	31.76	27.25
609.60	15.90	511.5	600.0	31.72	31.64
609.60	15.90	501.2	581.0	30.20	31.01
912.00	19.00	517.1	559.0	24.85	25.41
912.00	19.00	457.8	546.0	23.11	22.50
912.00	19.00	508.8	604.0	25.80	25.00
912.00	19.00	426.7	578.0	23.17	20.97
591.80	18.20	636.0	645.0	41.76	46.62
591.20	18.90	563.0	589.0	37.68	42.95
591.20	18.90	607.0	630.0	40.79	46.31
893.70	22.50	526.0	608.0	27.93	31.38
162.20	9.80	602.0	776.0	86.60	89.52
397.60	13.50	364.0	523.0	36.50	29.56
390.80	12.80	807.0	869.0	59.60	63.13
179.40	8.94	468.8	737.7	77.70	56.83
90.35	6.50	696.3	751.4	119.27	124.90
198.20	14.60	903.1	992.7	173.80	166.20
179.50	13.30	834.2	903.1	152.29	154.50
180.30	10.40	613.6	723.8	92.17	86.85
179.10	10.30	848.0	916.9	118.51	114.10
247.10	9.86	641.1	717.0	61.08	61.50
252.40	13.50	606.7	703.2	81.56	79.26
89.00	14.40	606.7	730.8	294.65	273.89
67.30	3.91	689.4	834.2	113.34	98.33
179.60	12.01	779.0	896.2	136.09	129.14
198.90	14.70	903.1	992.7	171.66	166.80
180.60	14.90	903.1	992.7	178.55	188.05

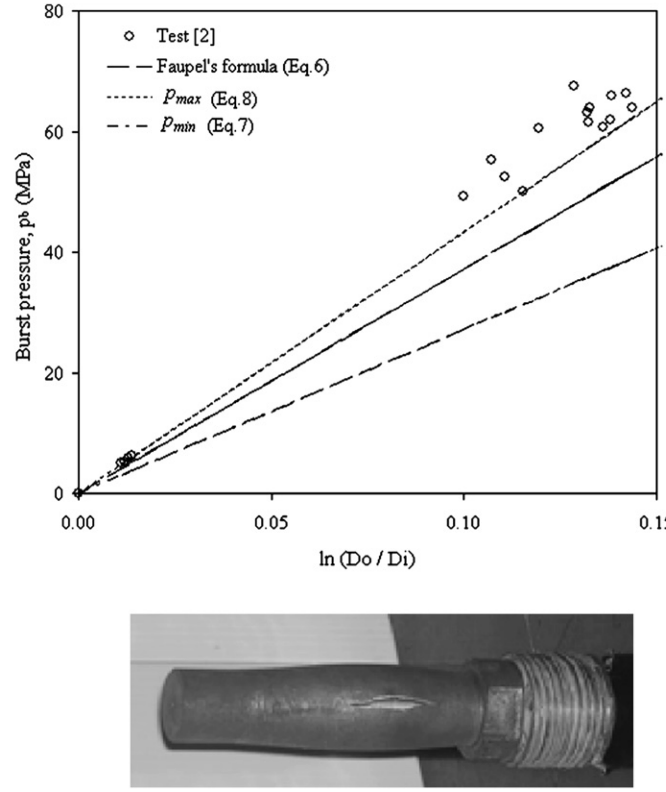

Fig. 3. Comparison of failure pressure estimates of Q235 (Gr.D) mild steel vessels with test data. A vessel of 250 mm length (excluding the screw thread part) indicating high plastic deformation after burst test.

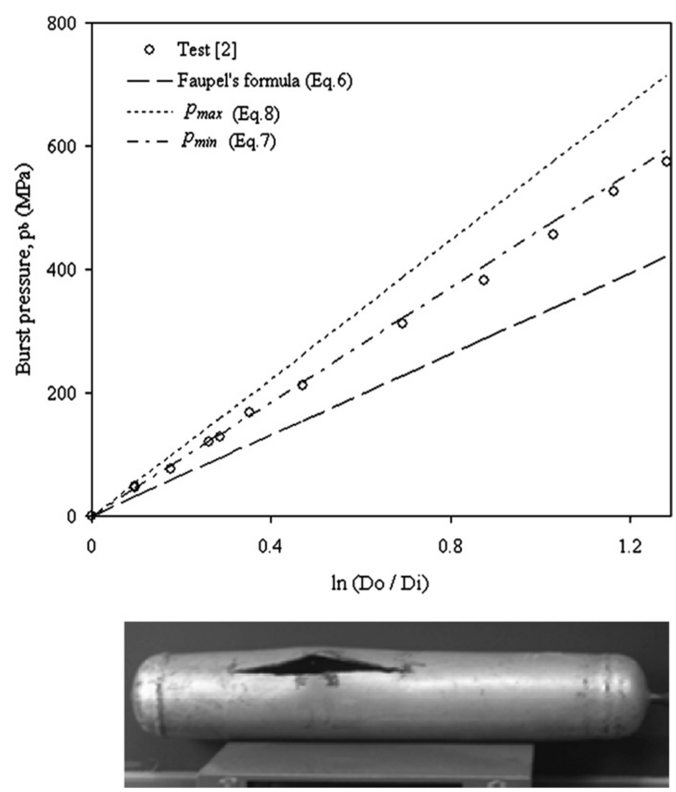

Fig. 4. Comparison of failure pressure estimates of 20R (1020) mild steel vessels with test data. A vessel of 500 mm length indicating high plastic deformation after burst test.

Table 3
Comparison of failure pressure estimates with test results of Q235 (Gr.D) and 20R (1020) mild steel cylindrical vessels.

Q235 (Gr.D) mild steel $\sigma_{ys} = 235$ MPa; $\sigma_{ult} = 375$ MPa			20R (1020) mild steel $\sigma_{ys} = 285$ MPa; $\sigma_{ult} = 484$ MPa		
$\frac{D_o}{D_i}$	Burst pressure, P_b (MPa)		$\frac{D_o}{D_i}$	Burst pressure, P_b (MPa)	
	Test [2]	Equation (8)		Test [2]	Equation (6)
1.105	49.20	43.23	1.102	47.80	45.10
1.116	55.20	47.52	1.102	47.60	45.10
1.117	52.40	47.91	1.102	45.10	45.10
1.122	50.00	49.84	1.192	76.03	81.56
1.127	60.50	51.77	1.300	119.68	121.84
1.134	67.50	54.45	1.330	128.32	132.43
1.139	66.80	56.36	1.422	167.26	163.49
1.141	63.20	57.11	1.600	212.39	218.26
1.142	61.60	57.49	2.000	311.85	321.89
1.142	64.00	57.49	2.400	381.48	406.55
1.146	60.80	59.01	2.800	456.90	478.14
1.148	62.00	59.76	3.200	526.62	540.15
1.150	66.00	60.52	3.600	574.69	594.85
1.153	66.40	61.65			
1.155	64.00	62.40			
1.014	6.28	6.02			
1.013	5.83	5.59			
1.012	5.32	5.17			
1.011	5.12	4.74			

and hence the failure pressure estimates based on Faupel's formula (6) are slightly higher than the test results.

Figs. 3 and 4 show a comparison of failure pressure estimates of mild steel cylindrical vessels with test results [2]. The vessels after the burst test, shown in Figs. 3 and 4, indicate high plastic deformation. The yield strength (σ_{ys}) and the ultimate tensile strength (σ_{ult}) of Q235 (Gr.D) mild steel are 235 and 375 MPa respectively. The test data of Q235 (Gr.D) mild steel cylindrical vessels in Fig. 3 and Table 3 are found to be higher than the P_{max} estimates and hence Faupel's bursting pressure formula (6) gives a failure pressure lower than the test results. Zheng and Lei [2] reported that the average error in Faupel's bursting pressure formula on the test data is 20% and provided an empirical relation for the burst pressure of mild steel cylindrical pressure vessels: $P_b = 13.21\sigma_{ys}(\frac{\sigma_{ys}}{\sigma_{ult}})^4\ln(\frac{D_o}{D_i})$. The test data in Fig. 3 is related to 20R (1020) mild steel cylindrical pressure vessels. The yield strength (σ_{ys}) and the ultimate tensile strength (σ_{ult}) of 20R (1020) mild steel are 285 and 484 MPa, respectively. The test data in Fig. 4 are found to be within the bounds of the expected P_{min} and P_{max} values from equations (7) and (8). Hence Faupel's bursting pressure formula (6) gives failure pressures close to the test results (see Table 3). The discrepancy in the predictions from Faupel's bursting pressure formula (if any) may be due to variations in the strength properties of the vessel material. There is no guarantee that the above empirical relation of Zheng and Lei [2] will be suitable for all mild steel cylindrical vessels.

4. Concluding remarks

Several predictive equations are compared with failure data of different steel vessels. Faupel's bursting pressure formula provides the failure pressure of cylindrical vessels close to the test results. However there is no single failure criterion which can predict accurately all failure pressures. The discrepancy in the predictions (if any) may be attributed to variations in the strength properties of the vessel materials.

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