

# Prediction of birefringence distribution for optical glass lens

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## Abstract

Based on the viscoelastic theory, simulations were carried out on glass-lens forming processes. In addition, the birefringence distribution was calculated by stress-optic relation. First, to verify this viscoelastic simulation, FEM analysis was performed with the same material properties and process conditions used in the Bruckner's experiment. The result of the analysis was compared with Nadai's and Bruckner's results. The simulation results were in good agreement with the deformation and birefringence distribution in the experimental results. Finally, the FEM simulation technique was applied to actual LD-lens. The simulation was performed for each different cooling rate and the results were compared to each other in terms of birefringence. To investigate the effect of initial preform shape to optical property, a simulation was performed by using the new-proposed-shape-preform and the results were compared to the present shape in terms of birefringence.

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## 1. Introduction

Following remarkable advancement in the IT and display industry and the high integration of mechanical components in the optical device, the glass product, which has demonstrated good optical performance and environmental reliability over plastics, is being used increasingly in the optic industry.

Due to no grinding or polishing process, the glass press forming method has advantage over conventional glass-forming method in terms of production rate and geometry of glass product [1].

In the press forming method, it is very important to avoid not only the occurrence of geometrical defects of final products, but also optical defects, such as birefringence. Birefringence is caused by optical anisotropy which originate from residual stress due to thermal history and deformation in glass-forming process [2]. Thus, it is also essential to predict the residual stress distribution of final product accurately.

Many works [3–5] have been published in the area of glass-forming simulation using computer-based analysis. Most of those researches have been performed using the viscous fluid model for conducting the simulation of conventional glass forming at high temperature. However, the glass press forming has

been performed in relatively low temperature, and the proper constitutive equations are required for a given temperature range. Glass is known to show viscoelastic behavior between the transition temperature and softening temperature, where forming and the first cooling stage occur.

In this paper, simulations of glass-lens forming processes were carried out based on the viscoelastic theory and were performed for each process stage: heating, pressing and cooling. The FEM simulation technique was verified with experiment and then applied to actual lens with each different cooling rate. Simulation results were compared to each other case in terms of birefringence.

## 2. Numerical modeling

The glass material usually behaves as an elastic solid at low temperatures and as Newtonian liquids at extremely high temperature. On the other hand, the forming and the first cooling processes are generally performed at intermediate temperatures and under moderate stresses where the mechanical behavior can be described with a linear viscoelastic law [6]. The viscoelastic constitutive equations, which are deduced from the Boltzmann superposition principle, are given as follows:

$$\sigma(t) = \int_{-\infty}^t G(t-s)\dot{\epsilon}(s)ds \quad (1)$$

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where  $G$  is called the relaxation modulus. Through the normalization method,  $G$  are given as follows:

$$g(t) := \gamma_{\infty} + \sum_{i=1}^N \gamma_i \exp \left[ \frac{-t}{\tau_i} \right] \quad (2)$$

Several contributions from literature show that most silica glasses are thermorheologically simple material over a range of temperatures near the glass transition [7]. So, the master curve is constructed by calculating the shift function.

Eq. (3) calculates the birefringence with stress distribution calculated from Eq. (1):

$$n_i - n_j = C(\sigma_i - \sigma_j) \quad (i, j = 1, 2, 3) \quad (3)$$

$n_i - n_j$  represents the difference of the orthogonal refractive index on the plane which is perpendicular to the light path. It represents the birefringence or path difference when the orthogonal rays pass through the unit thickness.  $i - j$  represents the difference in stress that is applied along the direction of the refractive index. Here, birefringence is calculated by multiplying stress-optic coefficient  $C$  [2].

### 3. Viscoelastic simulation of glass in simple compression case

#### 3.1. Introduction

Bruckner's simple compression experiment was achieved by MTS. The material property of soda-lime silica glass is shown in Table 1. The geometrical change of glass deformation at high temperature was observed and compared by Eq. (4) which is based on viscous flow model, and the birefringence was calculated experimentally from 1 mm-thick rectangular specimen observed by photo-elastic stress measurement [9]. In this case, the same glass was compressed by 30% with 0.02 mm/s and its cooling rate is 2 K/min:

$$r(t) = R \left( \frac{h_0}{h(t)} \right)^{0.75} \exp \left[ \frac{3}{8} \left( \frac{z^2}{h_0^2} - \frac{z^2}{h(t)^2} \right) \right] \quad (4)$$

In this study, two-dimensional axisymmetric simulations of simple compression were carried out with ABAQUS. To verify the viscoelastic simulation, FEM analysis was performed with the same material properties and process conditions used in Bruckner's experiment [8,9]. The specimen used was characterized by a cylindrical glass with a radius of 5 mm and a height of 10 mm. The result of the analysis was com-

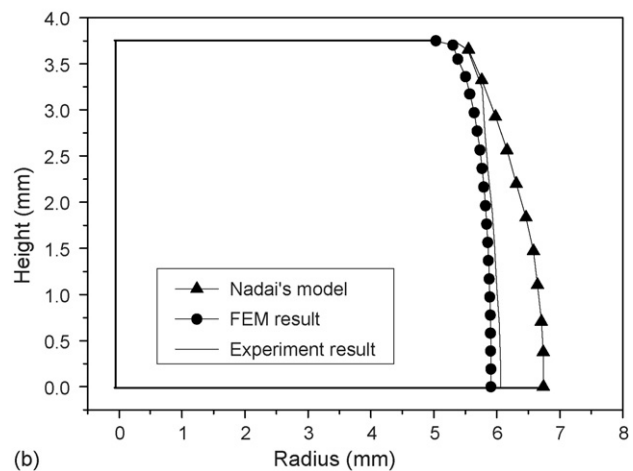
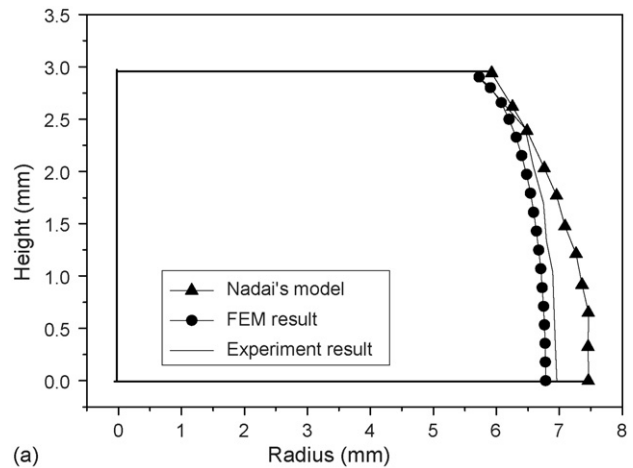


Fig. 1. Results of comparison data: (a) final shape after 42% reduction; (b) final shape after 35% reduction.

pared with the theoretical results and experimental results in terms of geometry of deformation and distribution of birefringence.

#### 3.2. Verification of FEM analysis by comparing the geometry

After 25% reduction with 0.02 mm/s and 42% reduction with 0.06 mm/s at 600 °C ( $10^{10}$  Pa s), the results from the FEM analysis were compared with the Bruckner's experimental results and with Nadai equation, respectively.

Fig. 1(a and b) illustrate that the tendency of overall change obtained by FEM analysis is close to the tendency achieved through the experiment. Furthermore, it shows that it is more accurate than one calculated using Nadai's equation. It is because Nadai's theoretical equation is based on the viscous flow model and many assumptions. In case of 25% reduction of specimen, the maximum radius obtained by FEM analysis was 5.91 mm, 6.06 mm by the experiment, and 6.77 mm by Nadai's equation. Compared to the experimental results, the error by FEM analysis was calculated to as 2.5%, and error by Nadai's equation was calculated as 11.7%. Therefore, the FEM analysis produced more

Table 1  
Material properties of soda-lime-silica glass

Mechanical property	Value	Thermal property	Value
Mass density	2.2 g/cm <sup>3</sup>	Thermal expansion coefficient	8.3E-06 °C
Poisson ratio	0.22	Heat capacity	840 J/Kg K
Elastic modulus	6.2E+01 GPa	Thermal conductivity	1.7 W/m K

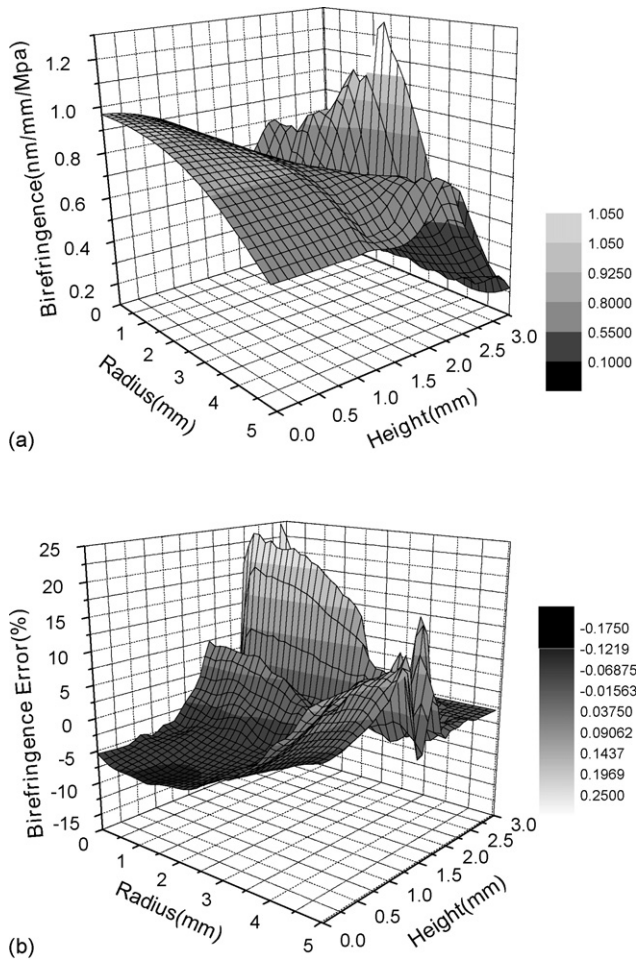


Fig. 2. Birefringence distribution: (a) birefringence distribution; (b) error of birefringence distribution.

reliable results and is more trustworthy in terms of geometrical match.

### 3.3. Verification of FEM analysis by comparing the birefringence distribution

The birefringence distribution, which is calculated using the residual stress distribution of the viscoelastic analysis using Eq. (1), was compared to Bruckner's experimental results. The photoelastic constant of silica glass was set as defined as 5 nm/mm MPa.

The result is shown in Fig. 2(a). Fig. 2(a) is a quarter part of the original specimen.

The comparison of the analytical result and the experimental result as shown in Fig. 2(b) shows a difference of two. The experimental value of birefringence at the center of specimen is 0.91, and the analytical value is 0.97 which means that there is 6.6% error. In addition, the error between experimental and analytical results is  $\pm 10\%$  in the center region, where the optical performance is critical. Due to error in the estimation for calculating boundary value of simulation results, however, the error in the outer region is about 10–20%.

Table 2

Material properties of die, glass, SUS304

	Die	Glass	SUS304
Thermal conductivity (W/m K)	45	0.862	16.2
Mass density (g/cm <sup>3</sup> )	15.4	4.49	8
Elastic modulus (GPa)	660	115.1	193
Poisson ratio	0.21	0.298	0.29
Thermal expansion coefficient ( $E-06^{\circ}\text{C}$ )	5	7	18.4
Heat capacity (J/kg K)	250	1000	500

## 4. Simulation of LD-lens

### 4.1. Introduction

LD-lens whose diameter is 1.7 mm, total diameter is 3 mm and height is 1.3 mm was used for this simulation. The lens was composed of glass and metal cylinder, which is made of SUS304 and covers the glass. The other properties are shown in Table 2. Table 3 shows weighting factor of glass in Eq. (2).

To model the LD-lens, four-node axisymmetric, thermo-mechanical coupled, quadrilateral elements were used. The interface heat transfer coefficient at the glass-die interface was assumed to be 2800 W/m<sup>2</sup> K [10]. The simulation was carried as follows: (1) Heating stage: heated up to 646 °C. (2) Compression stage: compressed at the rate of 0.1 mm/s at 646 °C (adiabatic). (3) First cooling stage: performed from forming temperature to glass transition temperature. (4) Second cooling stage: performed from glass transition temperature to 200 °C.

The effect of cooling rate and initial preform shape on the optical property of glass has been discussed. The first and second cooling rate of the present Insert Type LD lens was about 1 °C/s. Thus, the cooling rate of 1, 2, 4 and 8 °C/s was chosen and the birefringence distribution was obtained for each cooling rate. A ball-shaped preform was used for the entire deformation. The total element number of glass was 1737.

To investigate the effect of initial preform shape on the optical property, the simulation was carried out under the same condition as for the present LD-lens. The newly proposed-shape-preform has almost similar to that of the final shape of LD-lens. But the curvature of the lens showed slight differences. To assure that the same volume of lens was used, the width of preform was set as 0.845 mm and height as 1.310 mm. These values were obtained by calculating volume of lens.

### 4.2. Results of each cooling rate

Fig. 3 shows the residual stress distribution at each cooling rate after the cooling. As shown in Fig. 3, when the cooling rate

Table 3

Weighting factor of glass

Term no.	Weighting factor	Relaxation time
0	0.005	0
1	0.411	2.53
2	0.415	7.76
3	0.169	34.73

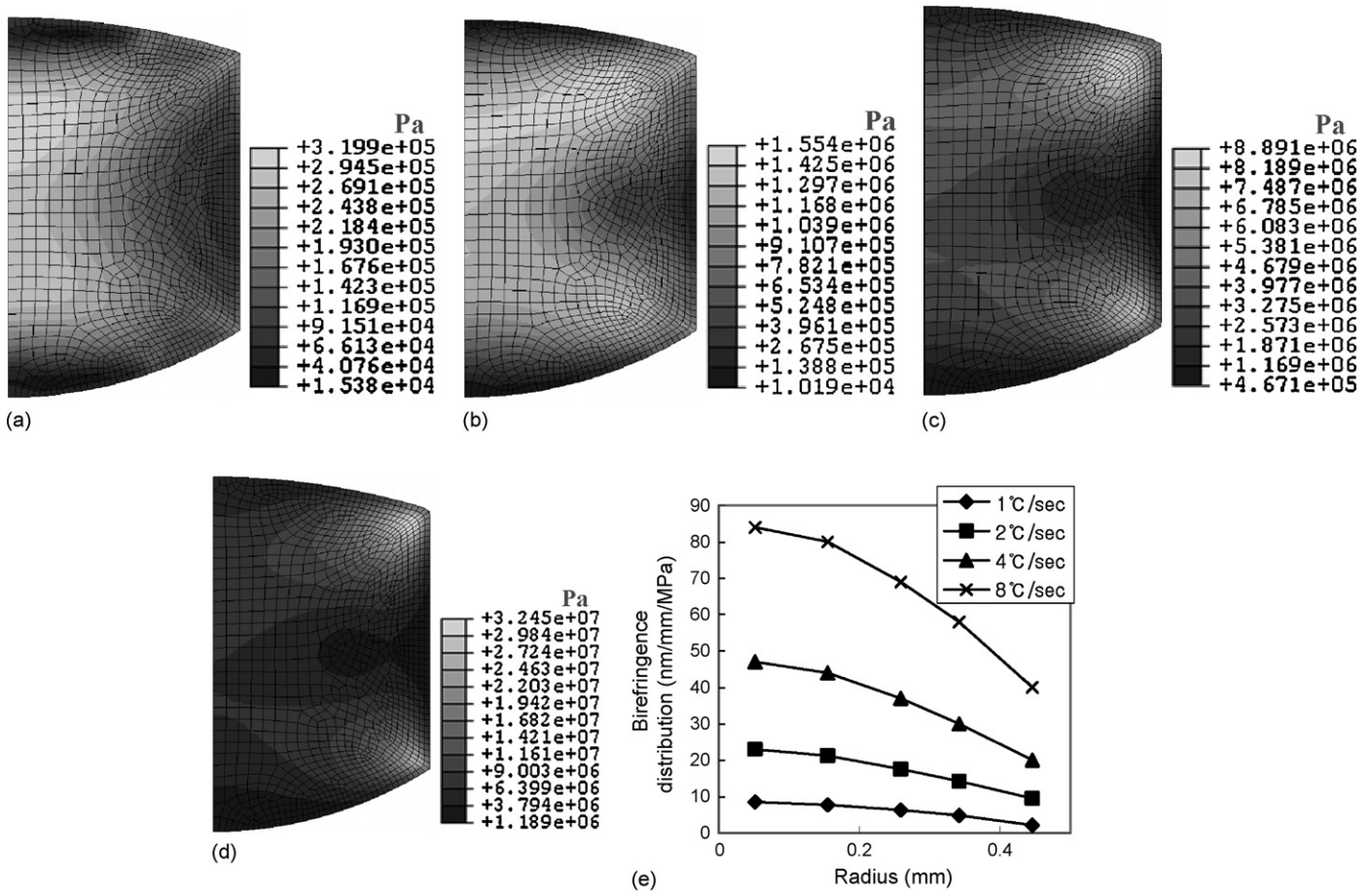


Fig. 3. Stress distribution at each cooling rate: (a) 1 °C/s, (b) 2 °C/s, (c) 4 °C/s, (d) 8 °C/s and (e) birefringence distribution.

was high, more residual stress was detected in the lens due to insufficient time. The distribution of birefringence up to 0.45 mm with each cooling rate is shown in Fig. 3(e). It can be seen that as the cooling rate increases, the losses by birefringence also increase.

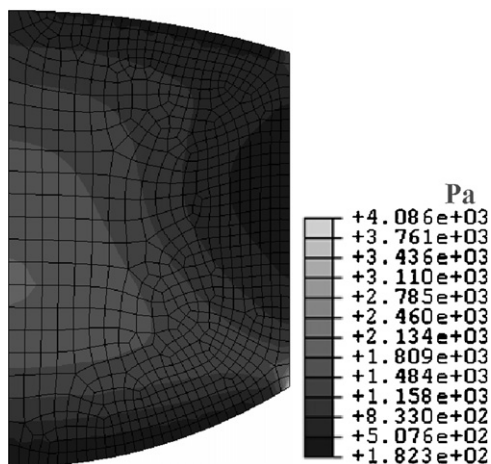


Fig. 4. Stress distribution in case of new preform.

#### 4.3. Results of initial preform shape

Fig. 4(a) shows the stress distribution of the simulation with the newly proposed-shape preform. As shown in Fig. 4(b), the value of birefringence compared to the simulation results with ball-shaped preform is shown at the same position. Due to very small deformation, the maximum residual stress in the lens is shown to be 0.004 MPa, and thus, the maximum value of birefringence is barely 0.02.

### 5. Conclusion

In this study, simulations of glass-lens forming processes were carried out based on the viscoelastic theory for each process stage: heating, pressing and cooling. In addition, the birefringence distribution was calculated by stress-optic relation.

- (1) The simulation results showed better agreement with Bruckner's experiment than the theoretical results provided by Nadai in terms of geometrical match. This simulation results were compared to Bruckner's measurement of birefringence. In terms of optical performance, the birefringence



distribution from the simulation was very similar to the experimental results.

- (2) The effects of cooling rate in the glass-forming of LD-lens were investigated in terms of optical performance derived from birefringence distribution.
- (3) When the shape of initial preform was changed, the birefringence distribution in the LD-lens was investigated. Present preform and new-proposed preform were compared in terms of birefringence distribution.

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