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The AdS_3 boundary energy–momentum tensor, exact in the string length over the curvature radius

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ABSTRACT

We clarify the map between boundary perturbations of AdS_3 in general relativity, and exactly marginal worldsheet vertex operators in AdS_3 string theory with Neveu–Schwarz Neveu–Schwarz flux. The latter correspond to solutions of the higher derivative low-energy tree level effective action to all orders in the string length over the curvature radius. We calculate the exact expression of the boundary energy–momentum tensor including all these higher derivative corrections in a purely bosonic string theory. The bottom-line is a canonical shift in the normalization of the boundary energy–momentum tensor corresponding to a shift in the curvature radius over the string length squared by the dual Coxeter number of the $SL(2,\mathbb{R})$ subalgebra of the space–time Virasoro algebra. That allows us to propose a value for the Brown–Henneaux central charge including all tree level higher derivative corrections in bosonic string theory, in a scheme dictated by the worldsheet conformal field theory.

1. Introduction

A quantum theory of gravity on AdS_3 with boundary conditions that allow for black hole configurations, has an asymptotic symmetry algebra which includes a (left and right copy of the) Virasoro algebra [1]. The algebra of diffeomorphism charges is centrally extended as shown through a Hamiltonian analysis in classical general relativity [1]. An alternative way to compute the central charge is through the holographic calculation of the Weyl anomaly [2].

String theory in AdS_3 is a consistent theory of quantum gravity that is likely to have a holographic dual. When we allow for black hole configurations in the bulk, the asymptotic Virasoro algebra is part of the symmetry algebra. Indeed, the space–time Virasoro algebra was explicitly constructed in terms of operators in the worldsheet conformal field theory in the case of AdS_3 backgrounds with Neveu–Schwarz Neveu–Schwarz flux [3–6], as well as in the case of backgrounds with Ramond–Ramond flux [7]. These constructions within the string worldsheet conformal field theory contain an infinite set of tree level higher curvature corrections to the classical general relativity analysis.

In this brief note, it is our aim to provide the necessary calculational details on the construction of the boundary energy–momentum tensor that takes into account all these higher derivative corrections in the context of bosonic AdS_3 string theory with pure NSNS flux. This allows us to speculate on an exact tree level value for the Brown–Henneaux central charge within an expansion scheme which is natural from the point of view of the worldsheet description of the string theory. Thus, we illustrate the power of an exact classical solution to string theory to determine all higher curvature corrections to a physical quantity, in the absence of supersymmetry.

2. The boundary energy–momentum tensor

In the holographic AdS/CFT correspondence [8–10] the conserved boundary energy–momentum tensor couples to the massless graviton in the bulk. In the first subsection we briefly review how we solve for the bulk deformation that corresponds to introducing a source term for the boundary energy–momentum tensor in the context of general relativity. In Sections 2.2 and 2.3, we show how to include all tree level higher derivative corrections in the context of string theory on AdS_3 with Neveu–Schwarz Neveu–Schwarz flux.

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2.1. Metric perturbations

We choose a gauge in which the radial components of the graviton at the boundary are zero, and we solve the bulk equations of motion for the graviton with boundary condition $\hat{h}_{ab}$ for the spatial components of the metric. For a two-dimensional boundary the perturbative solution to Einstein's equations with negative cosmological constant is given by [11,12]:

$$h_{\mu\nu}(x_0, x^i) = \frac{3}{\pi}\int d^2x' \frac{1}{|x-x'|^4} J_{\mu i}(x-x') J_{\nu j}(x-x') P_{ijab}\hat{h}_{ab}(x'_i), \tag{2.1}$$

where we work in the Poincaré coordinates with background metric:

$$ds^2 = \frac{1}{x_0^2}\left(dx_0^2 + dx_i^2\right) \tag{2.2}$$

and where we used the definitions:

$$P_{ijab} = \frac{1}{2}(\delta_{ia}\delta_{jb} + \delta_{ja}\delta_{ib}) - \frac{1}{2}\delta_{ij}\delta_{ab}, \qquad J_{\mu\nu}(x) = \delta_{\mu\nu} - \frac{2x_\mu x_\nu}{|x|^2}, \qquad \left|x-x'\right|^2 = x_0^2 + \left|x_i - x'_i\right|^2. \tag{2.3}$$

We use the conventions of [11] in which all the indices of the tensor P are contracted with the flat boundary metric. On the basis of this perturbative solution and the equality between the renormalized gravity action and the boundary generating functional of correlation functions, we can compute the boundary energy–momentum two-point function in the gravity approximation [11,12].

It will be useful to express the metric perturbation in terms of the following coordinates:

$$ds^2 = d\phi^2 + e^{2\phi}\, d\gamma\, d\bar{\gamma}. \tag{2.4}$$

When we concentrate on a source-term for the $\gamma\gamma$ component of the boundary-energy–momentum tensor only, we can write the bulk gravity solution after perturbation as:

$$\begin{aligned} h_{\bar{\gamma}\bar{\gamma}} &= \frac{3}{\pi}\int d^2\gamma \frac{e^{-4\phi}}{|x-\gamma|^8}\hat{h}_{\bar{\gamma}\bar{\gamma}}, \qquad h_{\gamma\gamma} = \frac{3}{\pi}\int d^2\gamma \frac{(\bar{\gamma}-\bar{x})^4}{|x-\gamma|^8}\hat{h}_{\bar{\gamma}\bar{\gamma}}, \qquad h_{\bar{\gamma}\gamma} = -\frac{3}{\pi}\int d^2\,\gamma e^{-2\phi}(\bar{\gamma}-\bar{x})^2\frac{1}{|x-\gamma|^8}\hat{h}_{\bar{\gamma}\bar{\gamma}}, \\ h_{\phi\gamma} &= \frac{6}{\pi}\int d^2\gamma \frac{e^{-2\phi}(\bar{x}-\bar{\gamma})^3}{|x-\gamma|^8}\hat{h}_{\bar{\gamma}\bar{\gamma}}, \qquad h_{\phi\bar{\gamma}} = \frac{6}{\pi}\int d^2\gamma \frac{e^{-4\phi}}{|x-\gamma|^8}(\bar{\gamma}-\bar{x}), \qquad h_{\phi\phi} = \frac{12}{\pi}\int d^2\gamma \frac{e^{-4\phi}(\bar{\gamma}-\bar{x})^2}{|x-\gamma|^8}\hat{h}_{\bar{\gamma}\bar{\gamma}}, \end{aligned} \tag{2.5}$$

where $\gamma, \bar{\gamma}$ are the complexified boundary coordinates and $|x-\gamma|^2 = e^{-2\phi} + (x-\gamma)(\bar{x}-\bar{\gamma})$ is a measure for the distance squared between a point in the bulk and one on the boundary.

2.2. Embedding in AdS_3 string theory

We want to generalize the above result to include all (tree level) higher derivative terms in the low-energy effective action in the context of bosonic string theory on AdS_3 with Neveu–Schwarz Neveu–Schwarz flux. To that end, we wish to recall the exactly marginal operator corresponding to the metric deformation reviewed in the previous subsection. The fact that we deal with a solvable worldsheet conformal field theory is the key to allow for an inclusion of all tree level higher derivative corrections.

We closely follow the paper [5] and refer to it for many useful concepts, technical details, as well as our notation. We recall that we have a left holomorphic worldsheet current algebra J^a as well as right anti-holomorphic current algebra $\bar{J}^a$, which can be conveniently packaged in x-dependent currents:

$$\begin{aligned} J(x;z) &= e^{-xJ_0^-} J^+(z) e^{xJ_0^-} = k\big((x-\gamma)^2 e^{2\phi}\partial\bar{\gamma} + 2(x-\gamma)\partial\phi - \partial\gamma\big), \\ \bar{J}(x;z) &= e^{-\bar{x}\bar{J}_0^-} \bar{J}^+(\bar{z}) e^{\bar{x}\bar{J}_0^-} = k\big((\bar{x}-\bar{\gamma})^2 e^{2\phi}\bar{\partial}\gamma + 2(\bar{x}-\bar{\gamma})\bar{\partial}\phi - \bar{\partial}\bar{\gamma}\big). \end{aligned} \tag{2.6}$$

We also introduce the primary vertex operator Φ_1 of worldsheet conformal dimension zero, and space–time conformal dimension one (which is also known as the bulk-to-boundary propagator for a massless scalar field):

$$\Phi_1 = \frac{1}{\pi}\frac{1}{(|\gamma - x|^2 e^{\phi} + e^{-\phi})^2}. \tag{2.7}$$

The derivative of the bulk deformation computed above in ordinary gravity with respect to the boundary metric component $\hat{h}_{\bar{\gamma}\bar{\gamma}}$ gives rise to a certain bulk deformation which corresponds to the naive boundary energy–momentum tensor. The metric components that code the bulk deformation, and therefore the boundary energy–momentum tensor, correspond to the following worldsheet vertex operator (as can be derived by combining equations (2.5), (2.6) and (2.7)):

$$T(x) \approx \frac{1}{2k}\int d^2z\left(J\partial_x^2\Phi_1 + 3\partial_x J\partial_x\Phi_1 + 3\partial_x^2 J\Phi_1\right)\bar{J}(\bar{x};\bar{z}). \tag{2.8}$$

The result of our canonical derivation agrees with the proposal of [5] to which we refer for many more interesting observations. In the worldsheet model, the worldsheet vertex operator is exactly marginal, and it is a physical vertex operator [5]. Our derivation of the vertex operator in the context of general relativity does not yet take into account possible higher derivative corrections to the boundary energy–momentum tensor $T(x)$, and we still need to accurately define the composite operator in the quantum theory on the worldsheet.

2.3. The space–time energy–momentum tensor

In this subsection, we wish to calculate the correct normalization of the boundary energy–momentum tensor, in the presence of all higher derivative terms coded in tree level string theory. An elementary argument for the need to modify formula (2.8) runs as follows. In bosonic string theory, when the worldshseet $SL(2)$ current algebra has level k, the worldsheet energy–momentum tensor has prefactor $1/(k-2)$ (see e.g. [13] for a review). The conformal dimensions of primary operators similarly have a $1/(k-2)$ dependence. Therefore, the bulk masses of states will be a function of $k-2$, and the boundary conformal dimensions as well. Thus, the space–time energy–momentum tensor is expected to depend on the level in the combination $k-2$.

That is one way to argue for the proposal for the exact boundary energy–momentum tensor:

$$T(y) = \frac{1}{2(k-2)} \int d^2z \left(\partial_x^2 \Phi_1 J + 3\partial_x \Phi_1 \partial_x J + 3\Phi_1 \partial_x^2 J\right) \bar{J}(\bar{x}; z). \tag{2.9}$$

In the above, we mean an exact equality between the (canonically normalized) boundary-energy–momentum operator and the normal ordered composite operator on the right (where the components of the composite operator are ordered as indicated). The normal ordering in the two-dimensional chiral conformal field theory is the one described for instance in [13], which consists of subtracting singularities in the OPE evaluated at the location of the second operator. We also introduce the operator:

$$C = -\frac{6}{k-2} \int d^2z \, (\Phi_1 J \bar{J})(z, \bar{z}) \tag{2.10}$$

and recall [5] that its derivatives with respect to either x or $\bar{x}$ decouple from all worldsheet correlation functions (since it is a total derivative operator without singularities when it encounters physical insertions on the worldsheet). Using this fact, we can write an equivalent expression for the boundary energy–momentum tensor:

$$T(x) = \frac{1}{2(k-2)} \int d^2w \left(\partial_x \Phi_1 \partial_x J + 2\Phi_1 \partial_x^2 J\right) \bar{J}(\bar{x}; z). \tag{2.11}$$

To prove that this is the canonically normalized boundary energy–momentum tensors to all orders in higher derivative corrections, i.e. to all orders in a $1/k$ expansion, it is sufficient to compute the operator product of the boundary energy–momentum tensor T with itself.

2.3.1. The TT operator product

An important operator equation in the following calculations is:

$$\partial \Phi_1 = \frac{1}{k-2} \partial_x (J\Phi_1), \tag{2.12}$$

which arises from the fact that the action of ∂ on the primary operator Φ_1 is equivalent to the action of the worldsheet Virasoro generator L_{-1} which can be computed via the worldsheet energy–momentum tensor which is the Sugawara bi-linear in the currents. The normalization of the energy–momentum tensor is $1/(k-2)$ for a level k current algebra.[1] Using a similar equation for deriving with respect to $\bar{x}$, we obtain the result:

$$\partial_{\bar{x}} T(x) = \frac{1}{2i} \oint dz \left(\partial_x \Phi_1 \partial_x J + 2\Phi_1 \partial_x^2 J\right). \tag{2.13}$$

From now on, we again follow [5] closely and compute the operator product $\partial_{\bar{x}} T(x) T(y)$. Our calculation is an interesting space–time analogue of the calculation of the shift in the level of the worldsheet energy–momentum tensor. Since it is a little intricate, we produce it here in some detail. Let's split up the calculation in several parts.

2.3.2. Preparation

We will make good use of the OPEs[2] between the currents and the primary fields:

$$\begin{aligned} J(x; z) J(y; z) &\sim k \frac{(x-y)^2}{(z-w)^2} + \frac{1}{z-w}\left((x-y)^2 \partial_y - 2(y-x)\right) J(y; w), \\ J(x; z) \Phi_h(y; w) &\sim \frac{1}{z-w}\left((x-y)^2 \partial_y + 2h(y-x)\right) \Phi_h(y; w), \end{aligned} \tag{2.14}$$

as well as all of their derivatives with respect to both x and y. These operator products code the chiral current algebra as well as the affine primary nature of the operator Φ_h of space–time dimension h [5]. Note that although the product $J(x; z)\Phi_h(y; w)$ is regular at $x = y$ [5], that is not true for various derivative operators. Composites of derivative operators therefore do require normal ordering. As stated previously, we take the derivative operators appearing in the proposed energy–momentum tensor (2.9) to be normal ordered in the order indicated. We also recall the regular operator product:

$$\Phi_1(x; z)\Phi_h(y; w) = \delta(x-y)\Phi_h(y; w) + O(z-w) \tag{2.15}$$

which was conjectured in [5] and proven in [14] (given our normalization of the vertex operators).

[1] Our equation differs from Eq. (4.15) in [5] at higher orders in the $1/k$ expansion. That leads to a canonically normalized space–time energy–momentum tensor. For the superstring, and worldsheet supersymmetric sigma-model discussed from Section 8 onwards in [5], their Eq. (4.15) becomes exact. In a purely bosonic context, it is a good semi-classical approximation.

[2] We use the conventions of [5] and refer to that reference for further definitions and details.

2.3.3. The space–time primaries

In order to show that the operator $\Phi_h(x;z)$ is indeed a space–time primary of dimension h, we wish to compute the operator product expansion $\Phi_h(x;z) \cdot \partial_{\bar{y}} T(y)$. The calculation below should be thought of as talking place inside a space–time correlation function. We compute the contribution due to the region where the operator T comes close to the operator Φ_h on the worldsheet. See [5] as well as [15] for a detailed discussion of why this is sufficient. We compute the operator product of composite operators via a point-splitting procedure and denote by $\lim_{:w' \to w:}$ the limit in which we subtract singularities as per the normal ordering. See [13] for a pedagogical discussion of this standard procedure. We compute:

$$\begin{aligned}
\Phi_h(x;z) \cdot \partial_{\bar{y}} T(y) &\sim \frac{1}{2i} \oint_z dw\, \Phi_h(x,z) \cdot \left(\partial_y \Phi_1 \partial_y J + 2\Phi_1 \partial_y^2 J\right) \\
&\sim \frac{1}{2i} \oint_z dw \frac{1}{w-z} \lim_{:w' \to w:} \left(\partial_y \Phi_1(y;w')\left(2(y-x)\partial_x - 2h\right)\Phi_h(x;w) + 2\Phi_1(y;w') 2\partial_x \Phi_h(x;w)\right) \\
&\sim \partial_{\bar{y}} \left(\frac{h}{(x-y)^2} \Phi_h(x;z) + \frac{1}{y-x} \partial_x \Phi_h(x;z)\right).
\end{aligned} \tag{2.16}$$

The result is consistent with the standard operator product for a space–time primary of dimension h:

$$T(x)\Phi_h(y) \sim \frac{h\Phi_h(y)}{(x-y)^2} + \frac{\partial_y \Phi_h(y)}{x-y}. \tag{2.17}$$

Note that this calculation by itself already fixes the normalization of the space–time energy–momentum tensor.

2.3.4. The current energy–momentum operator product

The second intermediate result we wish to compute is the operator product between the current and the (derivative of the) space–time energy–momentum tensor. We find:

$$\begin{aligned}
J(x;z) \cdot \partial_{\bar{y}} T(y) &\sim \frac{1}{2i} \oint_z dw \lim_{:w' \to w:} \partial_y \Phi_1(y;w') \cdot \left(\frac{2k(y-x)}{(z-w)^2} + \frac{1}{z-w}\left((x-y)^2 \partial_y^2 - 2\right) J(y;w)\right) \\
&\quad + \Phi_1(y;w') \cdot \left(\frac{4k}{(z-w)^2} + \frac{2}{z-w}\left(-2\partial_y J(y;w) + 2(y-x)\partial_y^2 J(y;w)\right)\right) \\
&\quad + \frac{1}{z-w'}\left(4(y-x)\partial_y + (x-y)^2 \partial_y^2 + 2\right)\Phi_1(y;w') \cdot \partial_y J(y;w) \\
&\quad + \frac{2}{z-w'}\left((x-y)^2 \partial_y + 2(y-x)\right)\Phi_1(y;w') \cdot \partial_y^2 J(y;w) \\
&\sim \pi\left(2\partial_y \Phi_1 J + 2\Phi_1 \partial_y J - 4(y-x)\partial_y \Phi_1 \partial_y J - 8(y-x)\Phi_1 \partial_y^2 J - 3(x-y)^2 \partial_y \Phi_1 \partial_y^2 J - (x-y)^2 \partial_y^2 \Phi_1 \partial_y J\right) \\
&\quad + \pi\, \partial\left(4(k+1)\Phi_1 + (2k+8)(y-x)\partial_y \Phi_1 + 2(x-y)^2 \partial_y^2 \Phi_1\right)(y;z).
\end{aligned} \tag{2.18}$$

Under the global $SL(2,\mathbb{R})$ charges (corresponding to a subgroup of the $SO(2,2)$ isometry group of the AdS_3 space–time), the space–time energy–momentum tensor $T(x)$ transforms as a tensor of weight two. That reasoning also determines the terms in the above expression that are not total derivatives on the worldsheet, i.e. the first two lines in the final result. We will use derivatives of the above operator product expansion in the following calculation.

2.3.5. The stress–energy tensor operator product

Finally, we present some details of the calculation of the operator product expansion of the (derivative) of the boundary energy–momentum tensor. We spit the operator appearing in the energy–momentum tensor $T(y)$ such that Φ_1 is at w' and J is at w, after which we take the normal ordered limit $\lim_{:w' \to w:}$. We calculate:

$$\begin{aligned}
\partial_{\bar{x}} T(x) T(y) &\sim \frac{1}{2(k-2)} \int d^2 w \lim_{:w' \to w:} \left(\partial_{\bar{x}} \partial_y \left(\frac{1}{(x-y)^2} \Phi_1(y;w') + \frac{\partial_y \Phi_1(y;w')}{x-y}\right) \cdot \partial_y J(y;w) \bar{J}(\bar{y})\right. \\
&\quad \left. + \partial_{\bar{x}} \left(\frac{1}{(x-y)^2} \Phi_1(y;w') + \frac{\partial_y \Phi_1(y;w')}{x-y}\right) \cdot 2\partial_y^2 J(y;w) \bar{J}(\bar{y})\right) \\
&\quad + \frac{2\pi}{2(k-2)} \int d^2 w \lim_{:w' \to w:} \left(\partial_y \Phi_1(w') \cdot \left((-k-4)\partial_x + 2(y-x)\partial_x^2\right)\partial \Phi_1(x) \bar{J}(\bar{y}) + \Phi_1(w') \cdot 2\partial_x^2 \partial \Phi_1(x) \bar{J}(\bar{y})\right. \\
&\quad + \partial_y \Phi_1(w') \cdot \left(2\partial_x \Phi_1 \partial_x J + 4\Phi_1 \partial_x^2 J + 3(x-y)\partial_x \Phi_1 \partial_x^2 J + (x-y)\partial_x^2 \Phi_1 \partial_x J\right) \bar{J}(\bar{y}) \\
&\quad \left. + \Phi_1(w') \cdot \left(-6\partial_x \Phi_1 \partial_x^2 J - 2\partial_x^2 \Phi_1 \partial_x J\right) \bar{J}(\bar{y})\right).
\end{aligned} \tag{2.19}$$

After using Eq. (2.12), the calculation reduces to a careful manipulation of identities for distributions (of the sort $\partial_x^n(g(x)\delta(x-y)) = g(y)\partial_x^n \delta(x-y)$). We find (the $\bar{x}$-derivative of) the end result:

$$T(x) \cdot T(y) \sim \frac{C}{2(x-y)^4} + \frac{2T(y)}{(x-y)^2} + \frac{\partial_y T(y)}{(x-y)}, \tag{2.20}$$

where the central charge operator C is equal to:

$$C = -\frac{6}{k-2}\int d^2w\, \Phi_1 J \bar{J}. \tag{2.21}$$

The operator product expansion is indeed of the canonical form, thus confirming the correct normalization of the boundary energy–momentum tensor, to all orders in the inverse radius (or inverse string tension) expansion.

A similar calculation shows that the central charge operator C is indeed central (up to an operator vanishing inside string correlation functions). Finally, the calculation provides us with the exact central charge operator C with prefactor $1/(k-2)$.

3. Final remark

In gravity, one can evaluate the central charge of the asymptotic Virasoro algebra(s) either by directly computing the algebra of charges corresponding to the asymptotic symmetry generators and then evaluating the central charge on the AdS_3 background [1,16], by computing the boundary energy–momentum tensor two-point function, or by computing the Weyl anomaly holographically [2]. There are contexts in which one can extend these calculations to include higher derivative corrections very effectively (see e.g. [17]) even without knowing all these terms exactly.

In the context of the conformal field theory description of AdS_3 string theory, we find a central charge operator C that can take different values in different states [5,6]. We can evaluate the vacuum expectation value of the operator C in the AdS_3 vacuum to obtain the central charge in that state. One can do this by the techniques developed in [14] and applied in [18].[3] We deduce the result that the normalized vev of the operator $\Phi_1 J\bar{J}$ is exactly -1. We must take note though that, as argued in [4], the leading contribution of two insertions of the energy–momentum tensor inside a correlation function comes from a disconnected diagram, where we factor out the energy–momentum two-point function. In order for the remaining factor to correspond to a normalized correlation function, we need the non-normalized two-point function of the energy–momentum tensor, i.e. the non-normalized vev of the operator C. We propose to take this into account in naive fashion by introducing a volume factor as a normalization factor. The volume is contained in the vacuum amplitude due to an integration over zero-modes, and it is mildly renormalized due to oscillator modes. The tree level contribution also carries a factor of g_s^{-2} where g_s is the string coupling constant. Therefore, we expect the normalization factor (up to numerical factors) to be $g_s^{-2}(k-2)^{3/2}V_C \approx l_s G_N^{-1}(k-2)^{3/2}$ where V_C is the volume of the extra compact directions (in string units), $l_s = \sqrt{\alpha'}$ is the string length, and G_N is the three-dimensional Newton constant. The factor $(k-2)^{3/2}$ corresponds to the renormalized volume of AdS_3. We take a definition of the three-dimensional Newton constant G_N that incorporates possible α'-corrections to the compactified volume.[4] We find therefore that the central charge operator C will contribute with a factor of $-\frac{1}{k-2}(-1)(k-2)^{3/2}l_s/G_N = (k-2)^{1/2}l_s/G_N$ to tree level amplitudes with two insertions of the boundary energy–momentum tensor. The overall k-independent coefficient can be fixed by comparing to the semi-classical gravity limit. In conclusion, in this scheme associated to the factorized worldsheet string theory, the exact central charge will be $\frac{3}{2}\sqrt{k-2}\frac{l_s}{G_N}$. That agrees again with a shift of the semi-classical space–time radius squared k by the dual Coxeter number of the $SL(2,\mathbb{R})$ subgroup of the space–time Virasoro algebra.

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[3] One uses the Ward identities computed in Appendix A of [18] and applies them to the operator $\Phi_1 J\bar{J}$ inserted in a two-point function of two unit operators.

[4] For instance, if we imagine an $AdS_3 \times S^3 \times T^{18}$ solution to bosonic string theory, we would have a derivative corrected volume of $2\pi^2(k+2)^{3/2}l_s^3$ for the three-sphere which goes into the definition of the three-dimensional Newton constant. The assumption on the AdS_3 string theory that we use here and throughout the Letter is that the worldsheet conformal field theory is factorized.