Physics Letters B 685 (2010) 229–234

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Bouncing universe and reconstructing vector field

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ARTICLE INFO

Article history:
Received 26 March 2009
Received in revised form 19 January 2010
Accepted 27 January 2010
Available online 1 February 2010
Editor: S. Dodelson

Keywords:
Massive vector field
Bouncing
Reconstruction
Parametrization

ABSTRACT

Motivated by the recent works of Golovnev et al. (2008), Chiba (2008) [1,2] where a model of inflation has been suggested with non-minimally coupled massive vector fields, we generalize their work to the study of the bouncing solution. So we consider a massive vector field, which is non-minimally coupled to gravity. Also we consider non-minimal coupling of a vector field to the scalar curvature. Then we reconstruct this model in the light of three forms of parametrization for dynamical dark energy. Finally we simply plot reconstructed physical quantities in a flat universe.

1. Introduction

Nowadays it is strongly believed that the universe is experiencing an accelerated expansion. The observation data confirm it as type Ia supernovae [3] associated with large scale structure [4] and Cosmic Microwave Background anisotropies [5] have provided main evidence for this cosmic acceleration. In order to explain why the cosmic acceleration happens, many theories have been proposed. The standard cosmological model (SCM) furnishes an accurate description of the evolution of the universe. In spite of its success, the SCM suffers from a series of problems such as the initial singularity, the cosmological horizon, the flatness problem, the baryon asymmetry and the nature of dark energy and dark matter, although inflation partially or totally answers some of these problems. Inflation theory was first proposed by Guth in 1981 [6]. Inflation is a period of accelerated expansion in the early universe. It occurs when the energy density of the universe is dominated by the potential energy of some scalar field called inflaton. Currently all successful inflationary scenarios are based on the use of weakly interacting scalar fields. Scalar fields naturally arise in particle physics including string theory and these can act as candidates for dark energy. So far a wide variety of scalar field dark energy models have been proposed. These include quintessence [7], K-essence [8], tachyon [9], phantoms [10], ghost condensates [11] and so forth. Two main reasons for the use of scalar fields to explain inflation are natural homogeneity and isotropy of such fields and their ability to imitate a slowly decaying cosmological constant [1]. However, no scalar field has ever been observed, and designing models by using unobserved scalar fields undermines their predictability and falsifiability, despite the recent precision data. The latest theoretical developments (string landscape) offer too much freedom for model-building, so higher spin fields generically induce a spatial anisotropy and the effective mass of such fields is usually of the order of the Hubble scale and the slow-roll inflation does not occur [12]. Then an immediate question is, can we do Cosmology without scalar fields? The authors of [1,2] have shown that a successful vector inflation can be simultaneously surmounted in a natural way, and isotropy of the vector field condensate can be achieved even in the case of triplet of mutually orthogonal vector fields [13]. In spite of inflation success in explaining the present state of the universe, it does not solve the crucial problem of the initial singularity [14]. The existence of an initial singularity is disturbing, because the space–time description breaks down "there". Non-singular universes have been recurrently presented in the scientific literature. Bouncing model is one of them that was first proposed by Novello and Salim [15] and Melnikov and Orlov [16] in the late 70's. At the end of the 90's the discovery of the acceleration of the universe brought back to the front the idea that $\rho + 3p$ could be negative, which is precisely one of the conditions needed for cosmological bounce in GR, and contributed to the revival of non-singular universes. Bouncing universes are those that go from an era of acceleration collapse to an expanding era without displaying a singularity [17]. Necessary

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0370-2693/$ – see front matter
doi:10.1016/j.physletb.2010.01.071

conditions required for a successful bounce during the contracting phase, the scale factor $a(t)$ is decreasing, i.e., $\dot{a} < 0$, and in the expanding phase we have $\dot{a} > 0$. At the bouncing point, $\dot{a} = 0$, and around this point, $\ddot{a} > 0$ for a period of time. Equivalently in the bouncing cosmology the Hubble parameter H runs across zero from $\dot{H} < 0$ to $H > 0$ and $H = 0$ at the bouncing point. A successful bounce is required around this point.

The remainder of the Letter is as follows. In Sections 2 and 3, we will consider vector field action proposed in Refs. [1,2] and study the bouncing solution of this model. In Sections 4 and 5, we will reconstruct physical quantities for this model and also will plot the corresponding graphs. Finally we will apply three parametrizations and compare them for this model.

2. Vector field foundation

We consider a massive vector field, which is non-minimally coupled to gravity [1,2]. The action is given by

$$S = \int d^4x \sqrt{-g}\left(\frac{1}{16\pi G}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 U_\mu U^\mu + \frac{1}{2}\xi R U_\mu U^\mu\right), \tag{2.1}$$

where $F_{\mu\nu} = \partial_\mu U_\nu - \partial_\nu U_\mu$, and ξ is a dimensionless parameter for non-minimal coupling. We note that the non-minimal coupling of a vector field is the same as the conformal coupling of a scalar field in case $\xi = 1/6$. We adopt the FRW universe with the metric signature of $(-+++)$.

The equations of motion are given by

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G\Big[F_{\mu\alpha}F^{\alpha\nu} - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} + (m^2 - \xi R)U_\mu U_\nu - \frac{1}{2}g_{\mu\nu}(m^2 - \xi R)U_\alpha U^\alpha - \xi g_{\mu\nu}U_\alpha U^\alpha + \xi(\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box)U_\alpha U^\alpha\Big], \tag{2.2}$$

$$\nabla_\nu F^{\nu\mu} - m^2 U^\mu + \xi R U^\mu = 0, \tag{2.3}$$

where the right-hand side of Eq. (2.2) is the energy–momentum tensor of the vector field U_i. The variation of the action with respect to U_i yields the following equations of motion,

$$\frac{1}{a^2}\nabla^2 U_0 - \frac{1}{a^2}\partial_i \dot{U}_i - m^2 U_0 + \xi R U_0 = 0, \tag{2.4}$$

$$\ddot{U}_i + \frac{\dot{a}}{a}(\dot{U}_i - \partial_i U_0) - \partial_i \dot{U}_i + \frac{1}{a^2}\big(\partial_i(\partial_k U_k) - \nabla^2 U_i\big) + m^2 U_i - \xi R U_i = 0, \tag{2.5}$$

where a is the scale factor, the dot denotes the derivative with respect to the cosmic time and the summation over repeated spatial indices is satisfied. Considering the quasi-homogeneous vector field $(\partial_i U_\alpha = 0)$ and Eq. (2.4) imply $U_0 = 0$, so that from Eq. (2.5) we obtain

$$\ddot{U}_i + H\dot{U}_i - 6\xi\left(\dot{H} + 2H^2 + \frac{k}{a^2}\right)U_i + m^2 U_i = 0. \tag{2.6}$$

By using the acceleration relation $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$ we achieve

$$\dot{H} + H^2 = \frac{-4\pi G}{a^2}\left(2\dot{U}_i^2 - 4(1+6\xi)HU_i\dot{U}_i + 6\xi U_i^2 H^2 - m^2 U_i^2 - \frac{2k}{a^2}\xi U_i^2\right), \tag{2.7}$$

where $H = \frac{\dot{a}}{a}$, $R = 6(\dot{H}^2 + 2H^2 + \frac{k}{a})$ and $R_0^0 = \dot{H} + H^2$ are the Hubble parameter, the Ricci scalar and the first component of the Ricci tensor, respectively. As we know, a dynamical vector field has generally a preferred direction, and introducing such vector field may not be consistent with the isotropy of the universe. In fact, the energy–momentum tensor of the vector field U_μ has anisotropic components. However, the anisotropic part of the energy–momentum tensor can be eliminated by introducing a triplet of mutually orthogonal vector fields. In that case, we obtain the energy density ρ and the pressure p of the vector fields,

$$\rho = \frac{1}{a^2}\left[\frac{3}{2}\dot{U}_i^2 - 3(1+6\xi)HU_i\dot{U}_i + 9\xi U_i^2 H^2 + \frac{3}{2}m^2 U_i^2 - \frac{9k\xi}{a^2}U_i^2\right], \tag{2.8}$$

$$p = \frac{1}{a^2}\left[\frac{3}{2}\dot{U}_i^2 - 3(1+6\xi)HU_i\dot{U}_i + 9\xi U_i^2 H^2 - \frac{3}{2}m^2 U_i^2 + \frac{3k\xi}{a^2}U_i^2\right]. \tag{2.9}$$

Now introducing the change of variable $\phi_i = \frac{U_i}{a}$ (for more details see Ref. [1]), Eq. (2.4) changes to

$$\ddot{\phi}_i + 3H\dot{\phi} + \left(m^2 + (1-6\xi)\big(\dot{H} + 2H^2\big) - \frac{6\xi k}{a^2}\right)\phi_i = 0. \tag{2.10}$$

Then we consider $\xi = 1/6$ and obtain the basic equations of motion for a curved universe in terms of ϕ_i,

$$\ddot{\phi}_i + 3H\dot{\phi} + \left(m^2 - \frac{k}{a^2}\right)\phi_i = 0, \tag{2.11}$$

$$H^2 + \frac{k}{a^2} = 4\pi G\left(\dot{\phi}_i^2 + m^2\phi_i^2 - \frac{k}{a^2}\phi_i^2\right), \tag{2.12}$$

$$\dot{H} + H^2 = -4\pi G\big(2\dot{\phi}_i^2 - m^2\phi_i^2\big). \tag{2.13}$$

One can see where equations of motion of a vector field are reduced to minimally coupled massive scalar fields. So the energy density ρ and the pressure p for the vector fields are derived in terms of ϕ_i in case $\xi = 1/6$ in the form,

$$\rho = \frac{3}{2}\dot{\phi}_i^2 + \frac{3}{2}m^2\phi_i^2 - \frac{3k}{2a^2}\phi_i^2, \tag{2.14}$$

$$p = \frac{3}{2}\dot{\phi}_i^2 - \frac{3}{2}m^2\phi_i^2 + \frac{k}{2a^2}\phi_i^2. \tag{2.15}$$

Now we are going to consider behavior of the different values of the parameter ξ for the vector field. We solve numerically Eq. (2.6) for $K = 0, +1, -1$ which implies the flat, close and open universe respectively. Fig. 1 shows the graph of the vector field with respect to time in all cases of K. One can see where the vector field has oscillation behavior and the magnitude slowly decreases with respect to time evolution. Also by increasing the parameter ξ, the magnitude of the vector field will increase, but the period of oscillation is constant. We note that negative values of ξ are actually the same as in the above result.

As mentioned above we suggest the following solution for $U_i(t)$,

$$U_i(t) = \sqrt{A}e^{-\gamma t}\cos(mt + \theta), \tag{2.16}$$

where the parameter A describes the oscillating amplitude of the field with dimension of $[\text{mass}]^2$. Also A is a relation with the parameter ξ. This solution implies the damping magnitude of the oscillating vector field.

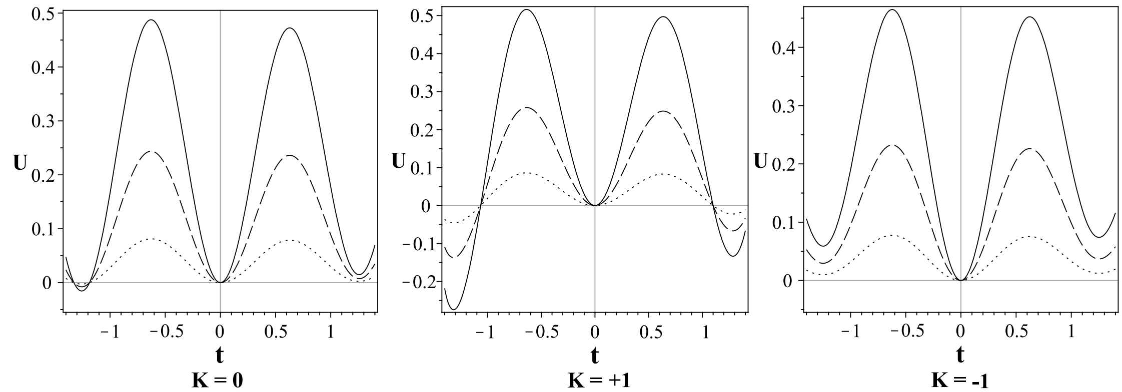

Fig. 1. Graphs of vector fields in terms of time. The solid, dash and doted lines represent $\xi = 1$, $\frac{1}{6}$ and 0.5 respectively.

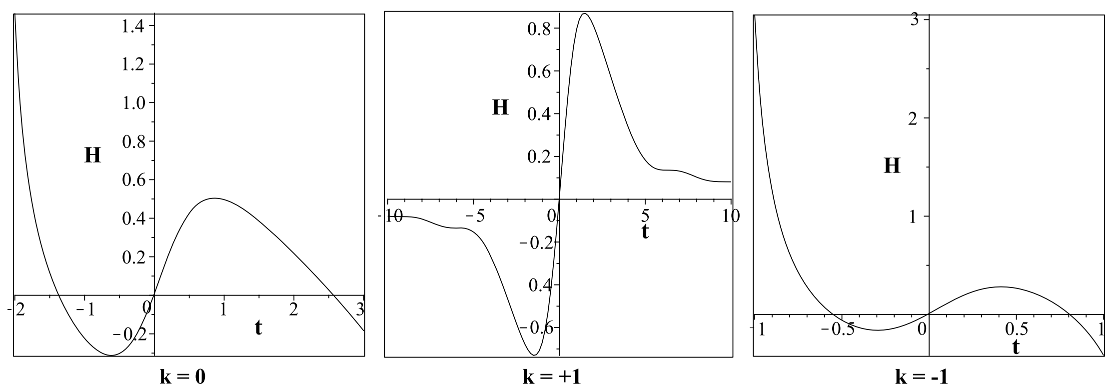

Fig. 2. The graphs of the Hubble parameter for $\xi = 1/6$, $4\pi G = 1$, $m = 1$ and $k = 0, +1, -1$ by choosing $\phi(0) = 1$, $\dot{\phi}(0) = 0.1$, $a(0) = 1$ and $H(0) = 0.01$.

3. Bouncing behavior

We will start with a detailed examination on the necessary conditions required for a successful bounce. During the contracting phase, the scale factor $a(t)$ is decreasing, i.e., $\dot{a} < 0$, and in the expanding phase we have $\dot{a} > 0$. At the bouncing point $\dot{a} = 0$ and around this point, $\ddot{a} > 0$ for a period of time. Equivalently in the bouncing cosmology the Hubble parameter H runs across zero from $H < 0$ to $H > 0$ and $H = 0$ at the bouncing point. A successful bounce is required around this point,

$$\dot{H} = -4\pi G(\rho + p) + \frac{k}{a^2} > 0. \tag{3.1}$$

At the point where the bounce occurs, Eqs. (2.8) and (2.9) reduce to

$$\rho_b = \frac{3}{2a^2}\left(\dot{U}_i^2 + m^2 U_i^2\right) - \frac{9k}{a^4}\xi U_i^2, \tag{3.2}$$

$$p_b = \frac{3}{2a^2}\left(\dot{U}_i^2 - m^2 U_i^2\right) + \frac{3k}{a^4}\xi U_i^2. \tag{3.3}$$

On the other hand, a successful bounce from Eqs. (2.6), (2.7) and (3.1) is obtained in the form,

$$\dot{U}_i^2 < \frac{1}{2} m^2 U_i^2 + \frac{k}{a^2}\xi U_i^2. \tag{3.4}$$

This result is similar to the slow-roll inflation. This means that one requires a flat potential which gives rise to a point bounce for the model of a vector field. From conditions (3.1), (3.4) it is clear that if we have bouncing solutions in an open universe, then we have such behavior for a flat and closed universe as well. Now we solve the above equation numerically for different values of ξ on the curved universe. Results are plotted in Fig. 2.

One can see the Hubble parameter H running across zero in any of three cases of k. In all cases of k, we have $H < 0$ to $H > 0$ which implies moving from a collapse era to an expanding era, and this result will not change for the different values of ξ in all cases of k. Also in Fig. 3, we can see the behavior of scale factor in terms of time for different values of k. It is clear that during the contracting phase, the scale factor $a(t)$ is decreasing, i.e., $\ddot{a} < 0$, and in the expanding phase we have $\ddot{a} > 0$, so the point where $\ddot{a} = 0$ is a bouncing point.

Therefore, in the vector field dominated universe we have a successful bouncing point in a close and flat universe but a turn-around point in an open universe. The bounce can be attributed to the negative-energy matter, which dominates at small values of a and creates a significant enough repulsive force so that a big crunch is avoided.

4. Reconstruction

Now we are going to present a reconstruction process for a vector field in the curved universe by $\xi = 1/6$. In this section, potential and kinetic energy are reconstructed with respect to redshift z. Also we obtain the EoS in terms of z. After that three types of parametrization are represented for the EoS. By using them we consider cosmology solutions such as the EoS, the deceleration parameter and vector field. The stability condition of this system is described by quantity of the sound speed. We rewrite Eqs. (2.14) and (2.15) in terms of the effective potential energy $\hat{V}$ and the effective kinetic energy $\hat{K}$ in the following form,

$$\rho = \frac{3}{2}\dot{\phi}_i^2 + \frac{3}{2}m^2\phi_i^2 - \frac{3k}{2a^2}\phi_i^2 = 3\hat{K} + 3\hat{V}, \tag{4.1}$$

$$p = \frac{3}{2}\dot{\phi}_i^2 - \frac{3}{2}m^2\phi_i^2 + \frac{k}{2a^2}\phi_i^2 = 3\hat{K} - 3\hat{V} - \frac{k}{a^2}, \tag{4.2}$$

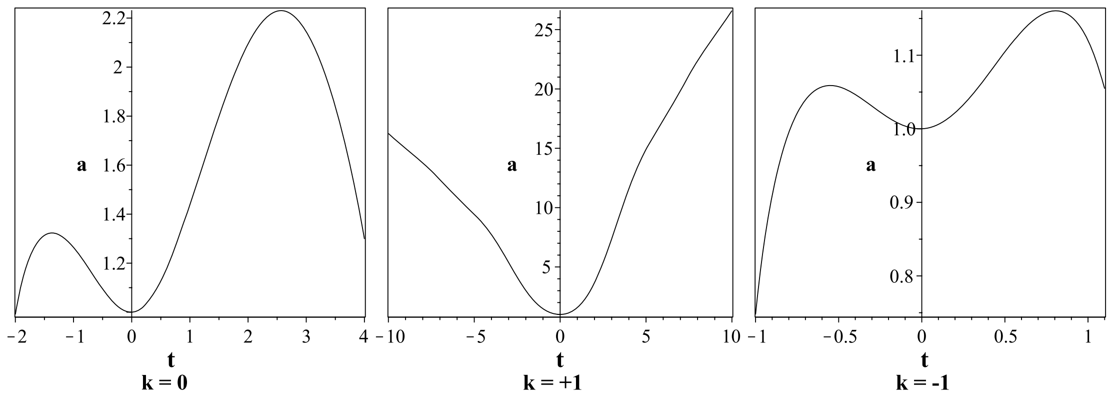

Fig. 3. The graphs of the scale factor for $\xi = 1/6$, $4\pi G = 1$, $m = 1$ and $k = 0, +1, -1$ by choosing $\phi(0) = 1$, $\dot{\phi}(0) = 0.1$, $a(0) = 1$ and $H(0) = 0.01$.

$$\rho + p = 6\hat{K} - \frac{k}{a^2}. \tag{4.3}$$

Then we can write the Friedmann equations as follows,

$$3M_p^2\left(H^2 + \frac{k}{a^2}\right) = \rho_m + \rho = \rho_m + 3\hat{K} + 3\hat{V}, \tag{4.4}$$

$$2M_p^2\left(\dot{H} - \frac{k}{a^2}\right) = -\rho_m - \rho - p = -\rho_m - 6\hat{K} + \frac{k}{a^2}, \tag{4.5}$$

where ρ_m is the energy density of dust matter. Also from Eqs. (4.1) and (4.2), we obtain relationship between the EoS with $\hat{V}$ and $\hat{K}$ as

$$\omega = \frac{p}{\rho} = \frac{3\hat{K} - 3\hat{V} - \frac{k}{a^2}}{3\hat{K} + 3\hat{V}} = -1 + \frac{2 - \frac{k}{3a^2\hat{K}}}{1 + \frac{\hat{V}}{\hat{K}}}. \tag{4.6}$$

We obviously have

$$\hat{V} + 3\hat{K} > \frac{k}{3a^2} \implies \omega > -1,$$
$$\hat{V} + 3\hat{K} < \frac{k}{3a^2} \implies \omega < -1,$$
$$\hat{V} + \hat{K} = \frac{k}{3a^2} \implies \omega = -1. \tag{4.7}$$

By using Eqs. (4.4) and (4.5) we can write

$$\hat{K} = \frac{-\rho_m}{6} - \frac{M_p^2}{3}\left(\dot{H} - \frac{k}{3a^2}\right) + \frac{k}{6a^2}, \tag{4.8}$$

$$\hat{V} = \frac{M_p^2}{3}\left(3H^2 + \dot{H} + \frac{2k}{a^2}\right) - \frac{\rho_m}{6} - \frac{k}{6a^2}. \tag{4.9}$$

Since in the present model, the dark energy fluid does not couple to the background fluid, the expression of the energy density of dust matter in respect of redshift z is [18],

$$\rho_m = 3M_p^2 H_0^2 \Omega_{m0}(1+z)^3, \tag{4.10}$$

where Ω_{m0} is the ratio density parameter of matter fluid and the subscript 0 indicates the present value of the corresponding quantity. By using the equation $1 + z = \frac{a_0}{a}$ (a_0 is quantity given at the present epoch) and its differential form we have

$$\frac{d}{dt} = -H(1+z)\frac{d}{dz}. \tag{4.11}$$

Introducing a new variable r as

$$r = \frac{H^2}{H_0^2}, \tag{4.12}$$

we rewrite the equation of motion of a vector field against z as

$$2r(1+z)^2U_i'' + 2r(1+z)\big(1 + H_0^2\big)U_i' - r'(1+z)^2U_i'$$
$$+ r'(1+z)U_i - rU_i + \frac{2m^2}{H_0^2}U_i - \frac{2k}{a_0^2H_0^2}(1+z)^2U_i = 0. \tag{4.13}$$

$\hat{K}$, $\hat{V}$ can be rewritten as follows,

$$\hat{K} = -\frac{1}{2}M_p^2H_0^2\Omega_{m0}(1+z)^3 + \frac{1}{6}M_p^2H_0^2r'(1+z)$$
$$+ \frac{k}{6a_0^2}(1+z)^2, \tag{4.14}$$

$$\hat{V} = M_p^2H_0^2r + \frac{2k}{3a_0^2}(1+z)^2 - \frac{1}{6}M_p^2H_0^2(1+z)r'$$
$$- \frac{1}{2}M_p^2H_0^2\Omega_{m0}(1+z)^3 - \frac{k}{6a_0^2}(1+z)^2. \tag{4.15}$$

By using Eqs. (4.6), (4.8) and (4.9) we obtain the following expression for the EoS,

$$\omega = \frac{(1+z)r' - 3r + \frac{k(1+z)^2}{a_0^2H_0^2M_p^2}(M_p^2 - 2)}{3r - 3\Omega_{m0}(1+z)^3 + \frac{k(1+z)^2}{a_0^2H_0^2M_p^2}(M_p^2 + 2)}. \tag{4.16}$$

Then we obtain the following equation for $r(z)$,

$$r(z) = \Omega_{m0}(1+z)^3 + (1 - \Omega_{m0})e^{\beta(z)}$$
$$+ \alpha_0 \int_0^z \big[\omega(\tilde{z})\big(2 + M_p^2\big) + \big(2 - M_p^2\big)\big](1+\tilde{z})e^{-\beta(\tilde{z})}\, d\tilde{z}, \tag{4.17}$$

where $\beta(z) = \int_0^z \frac{3w(\tilde{z})}{1+\tilde{z}}\, d\tilde{z}$ and $\alpha_0 = \frac{k}{a_0^2H_0^2M_p^2}$.

$$r(z) = \Omega_{m0}(1+z)^3 + (1 - \Omega_{m0})e^{3\int_0^z} \frac{1 + w(\tilde{z})}{1+\tilde{z}}\, d\tilde{z}. \tag{4.18}$$

Also we have the following expression for the deceleration parameter q,

$$q(z) = -1 - \frac{\dot{H}}{H^2} = \frac{(1+z)r' - 2r}{2r}. \tag{4.19}$$

Now we consider the stability of this model by using the hydrodynamic analogy and judge on stability by examining the value of the sound speed. Of course this is a simple approach, the perturbations in vector inflation are much richer than in the hydrodynamic

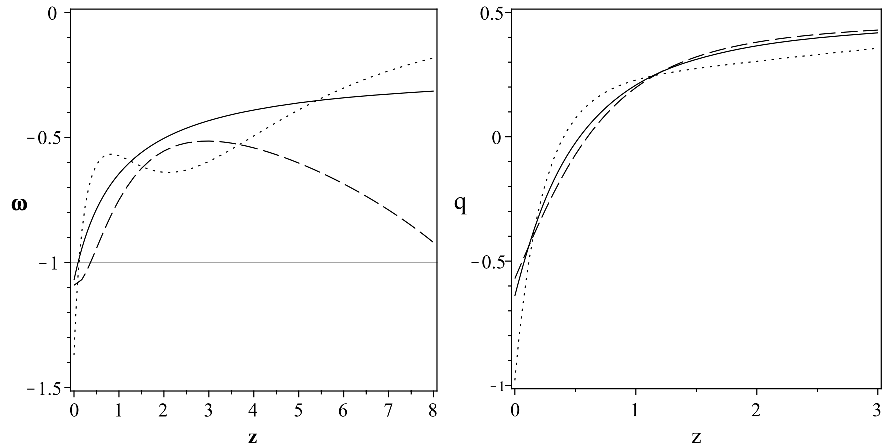

Fig. 4. Graphs of the EoS and deceleration parameter in respect of redshift z. The solid, dot and dash lines represent Parametrizations 1, 2 and 3, respectively.

model, see recent interesting works in [19,20]. The sound speed can be obtained by the following equation,

$$c_s^2 = \frac{p'}{\rho'} = \frac{-2r' + (1+z)r'' + 2\frac{k(1+z)}{a_0^2 H_0^2 M_p^2}(M_p^2 - 2)}{-9\Omega_{m0}(1+z)^2 + 3r' + 2\frac{k(1+z)}{a_0^2 H_0^2 M_p^2}(M_p^2 + 2)}. \tag{4.20}$$

In order to deal with the stability of our model, the sound speed must become $c_s^2 \geqslant 0$, so we can obtain from the above equation the following condition,

$$r(z) \geqslant \omega_{m0}(1+z)^3 - \frac{16ka_0^2}{H_0^2(1+z)^2}. \tag{4.21}$$

5. Parametrization

Now we consider the three different forms of parametrization as follows and compare them together.

Parametrization 1. First parametrization was proposed by Chevallier and Polarski [21], and Linder [22], where the EoS of dark energy in terms of redshift z is given by

$$\omega(z) = \omega_0 + \frac{\omega_a z}{1+z}. \tag{5.1}$$

Parametrization 2. Another the EoS in terms of redshift z was proposed by Jassal, Bagla and Padmanabhan [23] as

$$\omega(z) = \omega_0 + \frac{\omega_b z}{(1+z)^2}. \tag{5.2}$$

Parametrization 3. Third parametrization was proposed by Alam, Sahni, Saini and Starobinsky [24]. They take expression of r in terms of z as follows,

$$r(z) = \Omega_{m0}(1+z)^3 + A_0 + A_1(1+z) + A_2(1+z)^2. \tag{5.3}$$

By using the results of Refs. [25–27] and [28], we get coefficients of Parametrization 1 as $\Omega_{m0} = 0.29$, $\omega_0 = -1.07$ and $\omega_a = 0.85$, coefficients of Parametrization 2 as $\Omega_{m0} = 0.28$, $\omega_0 = -1.37$ and $\omega_b = 3.39$ and coefficients of Parametrization 3 as $\Omega_{m0} = 0.30$, $A_0 = 1$, $A_1 = -0.48$ and $A_2 = 0.25$. The evolutions of $\omega(z)$ and $q(z)$ are plotted in Fig. 4. Also, using Eqs. (4.14) and (4.15) and the three parametrizations, the evolutions of $\hat{K}(z)$ and $\hat{V}(z)$ are shown in Figs. 5 and 6 respectively. We note that the graphs can be simply presented only in the flat universe. From Figs. 4, 5 and 6, we can see that Parametrizations 1 and 3 are the same nearly and are slightly different from Parametrization 2. In Fig. 4, the EoS can fulfill the transition from $\omega < -1$ to $\omega > -1$ for any of three cases parametrization. Acceleration for all parametrizations shows tending to the positive value. $\hat{K}$ and $\hat{V}$ increase for Parametrizations 1 and 3, but in Parametrization 2 increase (decrease) is observed for $\hat{V}$ ($\hat{K}$). One can see that Parametrizations 1 and 3 satisfy the condition $\hat{K} + \hat{V} > 0$ and Parametrization 2 satisfies the condition $\hat{K} + \hat{V} = \text{const}$. By Eq. (4.7), we have $\omega > -1$ ($\omega = -1$) for Parametrizations 1 and 3 (Parametrization 2). This means that Parametrization 2 is better than other parametrizations.

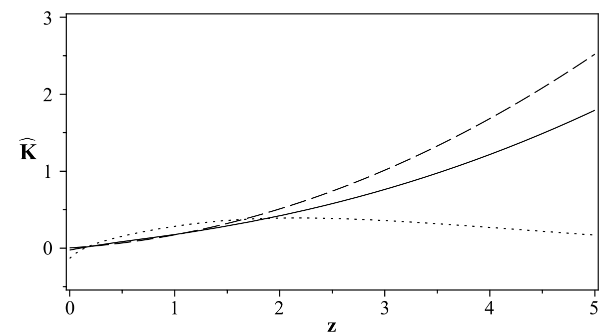

Fig. 5. Graphs of the reconstructed $\hat{K}$ in respect of redshift z. The solid, dot and dash lines represent Parametrizations 1, 2 and 3, respectively.

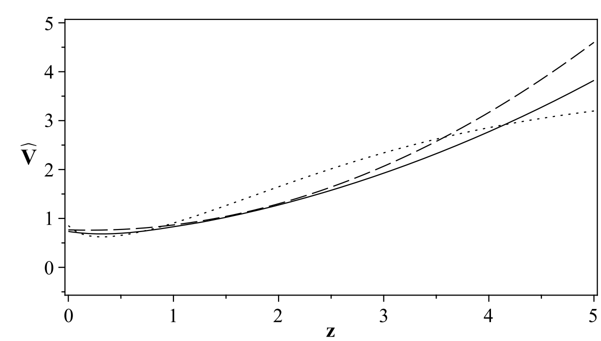

Fig. 6. Graphs of the reconstructed $\hat{V}$ in respect of redshift z. The solid, dot and dash lines represent Parametrizations 1, 2 and 3, respectively.

In Fig. 7, we can see the variation of the vector field against redshift z. This obviously shows slight difference between all parametrizations.

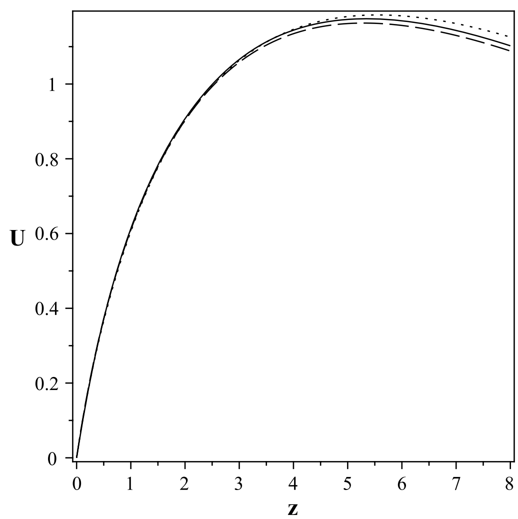

Fig. 7. Graphs of the reconstructed U_i in respect of redshift z. The solid, dot and dash lines represent Parametrizations 1, 2 and 3, respectively.

6. Conclusion

In this Letter, we have studied the bouncing solution in a curved universe which is proposed by the model of a massive vector field, U_i, non-minimally coupled to gravity. For our purpose we have derived the corresponding energy density, the pressure and the Friedmann equation for this model. Also we have obtained the bouncing condition as Eq. (3.4). From this condition, and also the essential condition (3.1), it is clear that if we have bouncing solutions in an open universe, then we have such behavior for a flat and closed universe as well. After we plot the Hubble parameter in terms of time in Fig. 2, for different k, we understood that our model predicts the bouncing behavior for all cases of k. From these figures one can see that the Hubble parameter H runs across zero in any of three cases of k. In all cases of k, we have $H < 0$ to $H > 0$ which implies moving from a collapse era to an expanding era, and this result will not change for the different values of ξ in all cases of k. After that in Fig. 3 we have shown that during the contracting phase, the scale factor $a(t)$ is decreasing, i.e., $\ddot{a} < 0$, and in the expanding phase we have $\ddot{a} > 0$, so the point where $\ddot{a} = 0$ is a bouncing point, and this figure is consistent with the results of Fig. 2.

After that we have investigated an interesting method as the reconstruction of the non-minimally coupled massive vector field model with the action (2.1). Our aim was to see whether the non-minimal coupling vector field can actually reproduce required values of observable cosmology, such as evolution of the EoS and the deceleration parameter in respect to the redshift z. We have reconstructed our model in three different forms of parametrization for the massive vector field. In Fig. 4 we have found the EoS crossing -1 in all parametrizations. The variations of reconstructed kinetic and potential energy against z have been plotted in Figs. 5 and 6, where in addition Parametrization 2 is better than two other parametrizations because $\hat{K} + \hat{V} = \text{const}$. Also we have investigated the stability of this system and have obtained a condition by the sound speed in all curvatures. Finally we note that reconstructed physical quantities have just executed in a flat universe and it is suggested for an open and close universe as future work.

Acknowledgements

The authors are indebted to the anonymous referee for his/her comments that improved the Letter drastically.

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