Available online at www.sciencedirect.com
ScienceDirect
International Journal of Forecasting 23 (2007) 115 – 126
international journal of forecasting
www.elsevier.com/locate/ijforecast

Non-linear forecasting of stock returns: Does volume help?

David G. McMillan *

School of Management, University of St. Andrews, The Gateway, North Haugh, St. Andrews, KY16 9SS, UK

Abstract

The testing for and estimation of non-linear dynamics in equity returns is a growing area of empirical finance research. This paper extends this line of research by examining whether a hitherto unconsidered variable, namely volume, imparts non-linear dynamics within equity returns and whether it has forecasting power. A significant amount of evidence supports a negative relationship between volume and future returns, which in turn suggests that volume could act as a suitable threshold variable. The results presented here provide evidence of a logistic smooth-transition model for four international stock market returns, with lagged volume as the threshold. Further, this model provides better out-of-sample forecasts than a corresponding logistic smooth-transition autoregressive model, a simple AR model and a random walk model based on a trading rule. In addition, this model also provides better forecasting performance in three cases against alternate non-linear specifications. This provides evidence in favour of non-linear dynamics, in contrast with previous evidence, which had suggested the relative failure of non-linear models in forecasting exercises.

JEL classification: C22; G12

Keywords: Stock market returns; Volume; LSTR model; Forecasting

1. Introduction

The testing for and estimation of non-linear dynamics in equity returns is a growing area of empirical finance research.[1] In particular, researchers have looked for the presence of univariate non-linearity, typically with daily data (for recent examples see Fang & Xu, 2003 and Shively, 2003). Where lower-frequency data has been employed, this has been largely discussed in the predictability literature, and thus variables such as the dividend yield, interest rates and output have been employed in the non-linear regression (see for example, Leung, Daouk, & Chen,

* Tel.: +44 1334 462800; fax: +44 1334 462812.
E-mail address: dgm6@st-andrews.ac.uk.

[1] This research has been conducted across a range of financial assets, including interest rate dynamics, for which major examples include Balke and Fomby (1997), Enders and Granger (1998) and Enders and Siklos (2001); exchange rate dynamics, Krägler and Krugler (1993), Obstfeld and Taylor (1997), and Coakley and Fuertes (2001); and equity returns, Martens, Kofman, and Vorst (1998), Perez-Quiros and Timmerman (2000), McMillan (2001, 2003) and Maasoumi and Racine (2002).

0169-2070/$ - see front matter © 2006 International Institute of Forecasters. Published by Elsevier B.V. All rights reserved.
doi:10.1016/j.ijforecast.2006.06.002

2000; Maasoumi & Racine, 2002).[2] The general consensus from this body of literature is that significant evidence of non-linear dynamics exists and that such models out-perform linear models in terms of in-sample diagnostics.

However, there exists less certainty over whether such models allow for improved out-of-sample forecasting (see, for example, De Gooijer & Kumar, 1992 for a general review and Brooks, 1997, for a specific application in the context of exchange rates). Several hypotheses have been advanced for the relative failure of non-linear models to outperform linear models in forecasting exercises. First, the continued use of linear forecasting performance metrics, where it has been argued that metrics should be more appropriate to the data and models used;[3] second, that the out-of-sample data period does not exhibit the same non-linear behaviour as the in-sample period, typically arising because out-of-sample periods have small time horizons; third, the non-linear model considered is in some way 'wrong' or not robust over the whole data period.

This paper seeks to extend this line of research by examining whether a hitherto unconsidered variable, namely volume, imparts non-linear dynamics within equity returns and whether it has sufficient forecasting power. There exists a substantial amount of evidence that a significant relationship exists between equity returns and volume. In particular, a sequence of relatively recent papers has argued that there exists a negative relationship between volume and future returns.[4] Most notably, Campbell, Grossman, and Wang (1993), Conrad, Hameed, and Niden (1994), Datar, Naik, and Radcliffe (1998), and Brennan, Chordia, and Subrahmanyam (1998) have all reported such evidence of a negative relationship between volume and returns. Furthermore, Wang and Chin (2004) have argued that low volume stocks are typified by momentum behaviour while high volume stocks exhibit reverting behaviour in returns. The rationale for such a negative relationship falls into two categories. First, market structure explanations such as those of Campbell et al. (1993) where informed traders acting as market makers use (high) volume as a signal that liquidity traders are active for non-fundamental reasons, and thus market makers adjust current prices according to the buying pressure to ensure adequate compensation for the risk; that is, if liquidity traders are selling, market makers will adjust current prices down to earn higher future returns. In a second market structure model, Blume, Easley, and O'Hara (1994) argue that by studying volume data, which provides information on the quality or precision of information contained in past price movements, traders can learn useful information about the course of prices, such that lagged volume has a significant relationship with current returns. The second set of explanations for the relationship between volume and returns stems from the recent behavioural finance literature. Barberis, Shleifer, and Vishny (1998) and Hong and Stein (1999) argue that momentum is consistent with low volume, as momentum arises from market underreaction due to insufficient updating of recently available public information (Barberis et al., 1998), or insufficient diffusion of information across 'news watchers' (Hong & Stein, 1999), and where low volume is a proxy for such informational lags. Baker and Stein (2002) argue that volume is an indicator of sentiment across irrational traders in the context of short sales constraints. More specifically, large volume for winner stocks would be consistent with irrational traders' trend-chasing behaviour, such that the stock price will rise above fundamental value before the subsequent reversion. However, a high volume for a loser stock is unlikely to arise from trend-chasing due to short sales constraints.

Therefore, given this negative relationship between volume and future returns, volume appears to be a natural transition variable for inclusion in a non-linear model. That is, when volume increases, or rises above a certain (threshold) level ('high volume') we would expect to see negative returns, and when volume is falling, or below a threshold level ('low volume'), we would expect to see positive returns. We therefore

[2] There is of course an extensive body of literature on linear predictability, for which examples include Balvers, Cosimano, and McDonald (1990), Cochrane (1991), Campbell and Hamao (1992), Ferson and Harvey (1993) and Pesaran and Timmermann (1995, 2000).

[3] See for example, Tiao and Tsay (1994), Tong (1995) and Clements and Smith (1999).

[4] Prior to these papers the major interest in examining stock market volume had been its relationship with the volatility of returns (see, for example, the review paper by Karpoff, 1987).

proceed to examine the relative forecasting performance of a non-linear model that includes volume against alternative non-linear specifications, a simple linear autoregressive (AR) model and a linear random walk model.

The remainder of this paper is organised as follows. Section 2 outlines the primary non-linear model considered, namely the logistic smooth-transition model. Section 3 presents both the empirical results and the results of the forecasting exercise. Section 4 considers some alternate non-linear specifications, while Section 5 provides a summary and concludes our findings.

2. Logistic smooth-transition model

From the preceding discussion, the nature of the relationship between equity returns and volume would appear to support a threshold-type model whereby the dynamics of equity returns alter between high and low volume states. As such we consider the logistic smooth-transition model (LSTR: Granger & Teräsvirta, 1993; Teräsvirta, 1994; Teräsvirta & Anderson, 1992). An alternative approach or model is the Heaviside TAR model (Tong, 1983). However, we choose the LSTR model because it is theoretically more appealing than the simple threshold model which imposes an abrupt switch in parameter values. This will only be the observed outcome when all traders act simultaneously; a smooth transition model is more appropriate for a market with many traders acting at slightly different times. The smooth-transition model also allows for the possibility of gradual movement between regimes, and therefore the potential for slow mean-reversion in asset returns (Campbell, Lo, & MacKinlay, 1997). Nevertheless, the LSTR model nests the TAR model as discussed below.[5]

The logistic smooth-transition model is given by:

$$r_t = \alpha_0 + \alpha_1 r_{t-1} + (\beta_0 + \beta_1 r_{t-1})F(x_{t-d}) + \varepsilon_t \tag{1}$$

where r_t is the return and $F(x_{t-d})$ is the transition function, with x_{t-d} being the transition variable, which is typically lagged returns, though in the present case this is extended to also include lagged volume. The logistic function is given as follows, with the full model therefore referred to as a Logistic STR (LSTR) model:[6]

$$F(x_{t-d}) = (1 + \exp(-\gamma(x_{t-d} - c)))^{-1} \tag{2}$$

where d is the delay parameter, γ the smoothing parameter, and c the transition parameter. This function allows the parameters to change monotonically with x_{t-d}. As $\gamma \to \infty$, $F(x_{t-d})$ becomes a Heaviside function $F(x_{t-d})=0$, $x_{t-d} \leq c$, $F(x_{t-d})=1$, $x_{t-d} \geq c$, and the model reduces to a threshold model (Tong, 1983). As $\gamma \to 0$, (1) becomes a linear model of order p.

3. Data and results

Daily national stock index and volume data were obtained from DataStream for the UK, US, France and Japan over the sample period 1/1/90 to 31/12/04. Daily data were chosen because, as was noted in the Introduction, one reason advanced for the failure of preferred in-sample non-linear models to report superior forecasting performances is the possibility that the out-of-sample portion of the data may not exhibit the same non-linear characteristics, which usually arises because the out-of-sample period is small. The belief being therefore, that it is more likely to find non-linear dynamics in the out-of-sample period for daily data. Following the convention we work with continuously compounded returns on each index, that is, we take the first-difference of the natural logarithm of the index. The volume data considered is the turnover (the number of constituent shares traded on the exchange on a particular day); this is consistent with that used by most previous studies, see for example Campbell et al. (1993) and Wang and Chin (2004) and references therein. In order to ensure stationarity of the volume variable we follow the convention in Campbell et al. (1993) and detrend the log turnover

[5] Furthermore, we also consider the performance of the TAR model in Section 4.

[6] Where a lag of the dependent variable is used as the transition variable this is typically referred to as a STAR (smooth-transition autoregressive) model.

Table 1
Descriptive statistics

Country	Stock returns				Detrended log volume				
	Mean	S.D.	Skew	Kurt	Mean	S.D.	Skew	Kurt	PP
UK	0.016	1.042	−0.092	6.081	6.2e−14	36.062	−1.112	8.852	−39.50
US	0.0326	0.984	−0.224	7.810	1.2e−13	37.722	0.298	3.463	−32.07
France	0.0225	1.175	−0.163	6.174	−4.1e−13	61.658	−0.917	4.589	−25.31
Japan	−0.0199	1.241	0.088	6.807	1.5e−13	69.310	−2.205	7.625	−57.48

Mean and S.D. values are in percentage form. The PP statistic is the Phillips–Perron test including a constant, with the bandwidth chosen by the Newey and West (1994) automatic selection method.

measure.[7] Summary statistics for each variable are presented in Table 1. These summary statistics reveal the usual characteristics of financial data, namely a mean value which is dominated by the standard deviation value and evidence of non-normality.

In order to conduct a forecasting exercise we use the period 1/1/90 – 31/12/99 (3915 observations) as the in-sample estimation period and the remainder of the sample, 1/1/00–31/12/04 (1305 observations), as the out-of-sample period. The estimation results for the AR model and the LSTR models both including and excluding volume (whole and sub-sample) are reported in Table 2.[8] The results for each model are largely consistent for both the whole sample and the sub-sample. A small positive autocorrelation is reported for the AR models. For the LSTR model with lagged returns as the transition variable (LSTAR model), positive autocorrelation is again typically found in both regimes (with the exception of Japan and the lower regime for the US sub-sample), which provides evidence of possible momentum or trending behaviour. For the LSTR model that has lagged volume as the transition variable, the lower regime is characterised by momentum, i.e., where the autoregressive parameters are positive, while the upper regime, determined by the sum of the autoregressive parameters, is characterised by random behaviour, or possible reversion (notably for France). These latter results are broadly consistent with those reported elsewhere (see the papers cited in the Introduction), and specifically with the results of Wang and Chin (2004) who state that low volume is associated with momentum behaviour, while high volume is associated with reverting behaviour in returns. Furthermore, these results are broadly consistent with the behavioural finance models of Barberis et al. (1998) and Hong and Stein (1999) who argue that momentum, arising from market underreaction, is consistent with low volume.

Having conducted the in-sample estimation, we proceed to examine the relative forecasting performance of the two non-linear models compared to those of the linear AR model and the random walk model. To gauge the ability of the models to forecast returns for each index in a straightforward way, we, in the first instance, conduct a simple series of one-step ahead forecasts and compute the root mean square error (RMSE), which are reported in Table 3. In accordance with many other studies, any forecast gain by the non-linear models over a simple random walk model is marginal. More specifically, for the UK and France the random walk model produces the lowest RMSE, while for the US and Japan the LSTR-volume model generates lower RMSE statistics but only at the level of the fourth decimal place, suggesting that any forecast gain is negligible.

However, as noted above, the RMSE may not be the best or most appropriate tool for gauging the forecasting power of competing models for financial data. That is, while the overall magnitude of the forecast error is important, it is perhaps of more relevance to practitioners to forecast the correct sign of price changes. For this reason, we proceed to further forecast evaluation. More specifically, to provide a further analysis of our

[7] Several detrending methods were considered including linear detrending and moving averages of various lengths. Again, following Campbell et al. (1993), the precise method of trend adjustment is less important that the need to adjust. This is borne out in the exercise conducted here where the results are qualitatively similar regardless of the detrending method.

[8] The choice of autoregressive lag in each regime was determined by parameter significance, while the use of $d=1$ for the transition variable was imposed to be consistent with only having a single autoregressive lag, but was also supported by formal tests, see Teräsvirta (1994).

Table 2
LSTR model estimates

Coefficients	Whole sample			Sub-sample		
	Linear	Transition function		Linear	Transition function	
		Returns (−1)	Volume (−1)		Returns (−1)	Volume (−1)
UK						
α_0	0.0003 (0.96)	0.0021 (1.86)	0.0006 (2.13)	0.0004 (2.12)	0.0021 (1.24)	0.0006 (2.03)
α_1	0.0013 (2.56)	0.0378 (2.01)	0.1143 (3.07)	0.0701 (2.85)	0.0791 (3.67)	0.1284 (3.75)
β_0		−0.0021 (−1.76)	−0.0007 (−1.91)		−0.0017 (−1.03)	−0.0003 (−0.84)
β_1			−0.1283 (−3.07)			−0.0904 (−2.10)
γ		822.2415 (0.31)	44.6217 (0.39)		716.1253 (1.10)	44.8666 (0.27)
c		−0.0209 (−11.62)	−0.1291 (−5.38)		−0.0206 (−5.82)	−0.1337 (−3.61)
US						
α_0	0.0003 (2.10)	0.0004 (2.18)	0.0006 (2.28)	0.0005 (3.05)	−0.0552 (−0.89)	0.0005 (2.30)
α_1	0.0036 (0.22)	0.0113 (0.68)	0.0822 (2.27)	0.0285 (1.46)	−0.3637 (−1.61)	0.1164 (3.49)
β_0		−0.0731 (−1.43)	−0.0005 (−1.35)		0.0720 (0.77)	−0.0001 (−0.22)
β_1		0.1952 (1.24)	−0.0992 (−2.44)		0.0174 (2.13)	−0.1377 (−3.30)
γ		28.5467 (0.40)	14.6363 (0.41)		0.5526 (1.45)	48.4382 (0.23)
c		0.0462 (48.47)	−0.0518 (−1.83)		−0.0221 (−1.85)	−0.0507 (−2.60)
France						
α_0	0.0002 (1.15)	0.0185 (0.01)	−1.54e−05 (−0.07)	0.0004 (2.15)	0.3225 (0.01)	−5.90e−06 (−0.03)
α_1	0.0394 (1.78)	0.1020 (0.62)	0.0531 (2.26)	0.0724 (2.57)	0.0918 (1.63)	0.0551 (1.83)
β_0		−0.0187 (−0.29)	0.0015 (2.87)		−0.3222 (−0.01)	0.0015 (2.84)
β_1			−0.1194 (−1.96)			−0.1187 (−1.89)
γ		0.5156 (0.32)	60.0383 (0.89)		1.1853 (0.29)	60.0383 (0.30)
c		−0.1715 (−0.02)	0.6085 (16.50)		−0.0900 (−0.09)	0.6091 (15.27)
Japan						
α_0	−0.0002 (−0.91)	−0.0206 (−1.32)	−0.0003 (−1.20)	−0.0001 (−0.51)	−0.0156 (−0.66)	−0.0002 (−0.83)
α_1	0.0892 (4.30)	−0.5482 (−1.79)	0.0945 (4.01)	0.0995 (3.54)	−0.5120 (−1.14)	0.0956 (2.78)
β_0		0.0245 (1.18)	0.0009 (1.25)		0.0154 (0.65)	0.0011 (1.60)
β_1		0.5477 (2.17)	−0.0735 (−2.27)		0.6375 (1.42)	−0.0840 (−2.73)
γ		0.9543 (1.73)	19.5836 (0.27)		2.9991 (0.27)	19.7895 (0.28)
c		−0.0207 (−1.92)	0.3672 (2.80)		−0.0430 (−8.16)	0.2437 (1.34)

Numbers in parentheses are White robust t-statistics. For model specification see Section 2; models are given by: Linear AR: $r_t=\alpha_0+\alpha_1 r_{t-1}\varepsilon_t$ LSTR: $r_t=\alpha_0+\alpha_1 r_{t-1}+(\beta_0+\beta_1 r_{t-1})F(x_{t-d})+\varepsilon_t$; $F(x_{t-d})=(1+\exp(-\gamma(x_{t-d}-c)))^{-1}$.

results, and to provide an additional evaluation method that may be of more interest to practitioners, we consider whether a simple trading rule based upon the estimated empirical models can provide an increase in trading returns over a buy-and-hold strategy. That is, whether a switching strategy based upon forecasts from the linear and non-linear models would allow investors to increase their returns.

Specifically, we conducted a recursive estimation approach where all parameters were re-estimated at each stage, including the threshold parameter, such that in effect, we are only allowing the information that would have been available to a trader in real time to enter into our estimation. First, we estimated our Eqs. (1)–(2) using information up to point t, where the threshold value is re-calculated using only information up to point t. We then use these parameter estimates to obtain a forecast of returns for the point $t+1$. As information from the time interval $t+1$ becomes available to a trader, we re-estimate all models and the threshold value up to this point and obtain a forecast for $t+2$. This process continues until the end of the sample is reached. At each point a decision is made whether to invest in the

Table 3
Forecasting evaluation

	Random walk	AR(1)	LSTR transition function		LSTR with forecasted volume
			Returns (−1)	Volume (−1)	
UK					
RMSE	0.012964	0.013036	0.013019	0.012992	
	Trading returns (%) – No short-selling				
Daily return	−0.0343	−0.0463	−0.0343	0.0101	0.0413
Cumulative	−45	−60	−45	13	54
	Trading returns (%) – Short-selling allowed				
Daily return	−0.0343	−0.0583	−0.0342	0.0155	0.0665
Cumulative	−45	−76	−45	20	88
US					
RMSE	0.012855	0.012868	0.012942	0.012649	
	Trading returns (%) – No short-selling				
Daily return	−0.0001	−0.0189	−0.0135	0.0223	0.0367
Cumulative	−0.13	−25	−18	29	48
	Trading returns (%) – Short-selling allowed				
Daily return	−0.0001	−0.0287	0.0179	0.0545	0.0885
Cumulative	−0.13	−37	−23	71	115
France					
RMSE	0.014413	0.014443	0.014471	0.014462	
	Trading returns (%) – No short-selling				
Daily return	−0.0254	−0.0361	−0.0332	−0.0001	0.0352
Cumulative	−33	−47	−43	−0.13	46
	Trading returns (%) – Short-selling allowed				
Daily return	−0.0254	−0.0469	−0.0411	0.0137	0.0938
Cumulative	−33	−61	−54	18	122
Japan					
RMSE	0.012956	0.012928	0.012890	0.012728	
	Trading returns (%) – No short-selling				
Daily return	−0.0331	0.0432	0.0456	0.0184	0.0001
Cumulative	−43	56	60	24	0.13
	Trading returns (%) – Short-selling allowed				
Daily return	−0.0331	0.1194	0.1242	0.0924	0.0490
Cumulative	−43	156	162	121	64

For a general discussion see Section 3. The RMSE is the root mean squared error. Trading returns are given by the following trading rules. The random walk is a buy-and-hold strategy. The strategy of the AR(1) model and LSTR model with returns as the transition variable is to buy if the next period forecast of returns is greater than zero, and to not buy (sell) if the next period forecast of returns is less than zero. For the models incorporating volume, when the volume (or forecast volume) is below its threshold value the strategy is to buy in the next period if forecast returns are positive in the current period, and to not hold (sell) if the forecast returns are negative. When the volume (or forecast volume) is above its threshold value the strategy is to not hold (sell) in the next period if the current forecast value is positive and to buy if the forecast value is negative.

index portfolio, or not to invest. These results are then compared with a strategy that invests in the index at the start date and continues to hold the index throughout the sample period.[9]

[9] This recursive process also allows time-variation in the threshold parameter (and indeed all parameter values). Plots of this time-variation are available for the interested reader upon request.

The decision criterion as to whether to purchase the portfolio is based on the forecasted returns. Specifically, for the LSTAR model (the model that uses returns as the switching variable), if the forecasted return is greater than 0 then the fund is purchased, otherwise not (we also compute the return allowing for short-selling, i.e., if the forecast value is negative sell the index fund). For the LSTR model with volume as a switching variable, when volume is below the threshold value we assume (following the estimation results and the results reported by Wang & Chin, 2004, as well as others cited in the Introduction) that returns exhibit continuation and hence buy the fund in the next period if returns are forecast positive in the current period, and do not hold (sell) if forecast negative. While if volume is above the threshold value we assume returns follow a reverting pattern and hence do not hold (sell) in the next period if the current forecast value is positive and buy if the forecast value is negative. We also extend this latter forecasting exercise by augmenting our LSTR model with an autoregressive model for volume. Thus, the above strategy is adjusted so that the decision on whether to follow a momentum- or reversion-type strategy depends upon whether the forecast value of the volume is above or below the threshold value.

The results from this exercise, which are presented in Table 3, reveal that for a buy and hold strategy the daily percentage return for each market is negative, as over the forecast horizon each index fell, resulting in a cumulative loss of 45%, 0.13%, 33% and 43% for the UK, US, France and Japan respectively. For the AR model the results are worse, with a cumulative loss of 60% (76%), 25% (37%), and 47% (61%) with no short-selling (short-selling allowed) for the UK, US and France, respectively. The exception to this is Japan where the AR model produces a trading profit.[10] For the LSTAR model (lagged with returns as the switching variable) the performance of the portfolio is not improved over the linear alternatives, for example, for the no short sales strategy the daily return is −0.03%, −0.01%, −0.03%, 0.05%, with a cumulative loss also of 45%, 18%, and 43%, and a gain of 60% for the UK, US, France and Japan, respectively. On the other hand, for the strategy which allows short-selling the daily loss is 0.03%, 0.02%, 0.04%, gain of 0.12% with a cumulative loss of 45%, 23%, 54%, gain of 162% for the UK, US, France and Japan, respectively. These results are typical in the sense that the non-linear model has not been able to produce superior forecasting performance to the linear model.

In contrast, a much improved portfolio performance is obtained for the model that uses volume as the switching variable for the UK, US and France. First for the forecasting model that uses actual volume, for the portfolio that does not allow for short-selling the daily return (cumulative return) is positive at 0.01% (13%), 0.02% (29%), −0.0001% (−0.13%) and with short-selling a return of 0.02% (20%), 0.05% (71%), 0.01% (18%) for the UK, US and France. For Japan a positive trading performance is still obtained, but it is lower than that reported for the previous LSTAR model. The performance is even further enhanced (again with the exception of Japan) when the forecasted volume is used as the decision variable in building the portfolio. Here there is a daily (cumulative) return of 0.04% (54%), 0.04% (48%), 0.04% (46%) for the UK, US and France, respectively, for the no short-selling portfolio and a return of 0.07% (88%), 0.09% (115%), 0.09% (122%) for the portfolio with short-selling for the UK, US and France, respectively. These results confirm that the use of volume in guiding portfolio decision-making can drastically improve the performance of the portfolio.

These last results suggest that when volume is incorporated into the forecasting model for returns the performance of the simple trading rule is significantly enhanced. Moreover, the best results, in terms of the highest return to an investor, are obtained where the LSTR non-linear model is augmented with a second model for forecasting volume (this is with the exception of Japan where the highest return is obtained using the LSTAR model). However, at this stage the reported returns are gross with no consideration of transaction costs yet being made. Thus, we reconsider our results for the most successful model. That is, we now include a transaction cost of 0.1%, 0.25% or 0.5% every time a switch in the portfolio is made for the LSTR-forecast volume model for the UK, US and France and the LSTAR model for Japan. These results reveal that the daily net returns are 0.04%, 0.01% and −0.05% for the UK in respect to the low, medium and high transaction cost; 0.06%, 0.02%, −0.05% for the US; 0.07%, 0.03% and −0.03% for France; and 0.05%, −0.02% and −0.12% for Japan. Thus, small transaction costs do not affect the nature of the result; that is, portfolio switching according to the appropriate model can lead to positive returns, although larger transaction costs do appear to negate any trading profits.

[10] The results for Japan differ to those of the UK, US and French markets; this appears to arise from the very strong autocorrelation identified in Table 2.

4. Alternate non-linear models

Whilst the LSTR model is our preferred specification for the reason stated in the Introduction, namely, the belief that the returns process differs between high and low volume states and the adjustment may be smooth and not instantaneous, we nevertheless allow for the possibility of alternate non-linear specifications. Thus, in this section we consider three of the more popular models, the threshold autoregressive (TAR) model, the momentum-TAR (MTAR) model and the exponential smooth-regression model (ESTR); in each case volume is used as the transition variable. The general specifications for the TAR and MTAR models are given by:

$$r_t = \alpha_0 + \alpha_1 r_{t-1} I_t + \beta_1 r_{t-1}(1-I_t) + \varepsilon_t \tag{3}$$

where the indicator variable I_t equals 1 if the (lagged) volume is above its threshold and 0 otherwise for the TAR model, while for the MTAR model the change in volume is used as the transition variable.[11] The ESTR model is given by:

$$\begin{aligned} r_t = {} & \alpha_0 + \alpha_1 r_{t-1} \\ & + (\beta_0 + \beta_1 r_{t-1})(1-\exp(-\gamma x_{t-1}^2)) + \varepsilon_t. \end{aligned} \tag{4}$$

This model implies that the dynamics of the middle ground differ from the dynamics of larger deviations, i.e., the returns process differs between small and large volume.[12] The estimation and forecasting results from these three models are presented in Tables 4 and 5.

For the purposes of exposition we briefly state the estimation results and concentrate on the forecasting performance of each model. In general, for each series the TAR and MTAR models typically support positive autocorrelation, while for the ESTR models the average volume regime is characterised by positive autocorrelation and a large non-average volume by negative autocorrelation. The forecast results of these three models are presented in Table 5. In comparing them with the results for the linear and LSTR models presented in Table 3, we can see that the RMSE statistics are of a similar magnitude. For the UK the RMSE is marginally lower for the ESTR model than that previously reported, while for the US and Japan the MTAR and TAR models provide a marginally superior RMSE performance. With regard to the trading rule, based evaluations, for the UK the TAR, MTAR and ESTR models perform poorly and obtain negative trading returns of similar magnitudes to those obtained by the AR and LSTAR models. The ESTR model with forecasted volume and where short-selling is allowed does produce a positive trading return but it is less than half that obtained by the equivalent LSTR model. Similarly, for the US, the TAR, MTAR and ESTR models all perform poorly in comparison to the LSTR model, with only the short-selling allowed MTAR model producing a trading profit. In contrast, however, the ESTR model with forecast volume does produce superior trading profits in comparison with the equivalent LSTR model. The results for France are akin to those of the UK and US in that the TAR, MTAR and ESTR models perform worse than the LSTR model, and, with the exception of the short-selling allowed MTAR model, produce trading losses. Only the ESTR model with volume forecasts produces positive trading profits, but as with the results for the UK, these are less than for the equivalent LSTR model. Finally, for Japan all models considered produce positive trading profits; however, none of these are substantially greater than those obtained for the AR and LSTAR models.

In summary, for the UK, US and France the TAR, MTAR and ESTR models typically produce negative trading returns and perform no better than the linear AR or LSTAR models. Only the ESTR model augmented with forecast volume produces positive trading returns, which for the UK and France are below the returns obtained for the equivalent LSTR model, while for the US the returns generated from the ESTR-forecast volume model are marginally higher than those of the

[11] In order to determine the threshold value in the TAR and MTAR models we follow the procedure in Chan (1993) in which we search over the potential threshold values so as to find the one that minimises the sum of squared errors from the fitted model and which has been shown to yield a super-consistent estimate of the threshold (Chan, 1993). More specifically, volume and the change in volume are sorted in ascending order, with the largest and smallest 15% of the values discarded. For each of the remaining possible threshold values, an equation of the form (3) is estimated, with the threshold yielding the lowest sum of squared residuals deemed to be the appropriate threshold value.

[12] At first glance this may appear to identify similar dynamics to the LSTR model, this is not the case. Recalling that the volume variable is detrended, this model identifies differences between average and non-average levels of volume, whereas the LSTR specification identifies below and above average volume.

Table 4
Alternate volume model estimates

Coefficients	UK			US		
	TAR	MTAR	ESTR	TAR	MTAR	ESTR
α_0	0.0004 (2.08)	0.0003 (1.97)	0.0007 (1.24)	0.0005 (3.08)	0.0005 (3.04)	0.0003 (0.44)
α_1	0.0457 (2.12)	0.1053 (4.13)	0.1798 (1.98)	0.0472 (1.88)	0.0187 (0.90)	0.2170 (2.36)
β_0			−0.0007 (−1.09)			0.0003 (0.42)
β_1	0.1796 (3.94)	0.0194 (0.64)	−0.2010 (−2.17)	0.0001 (0.35)	0.1106 (1.92)	−0.2098 (−2.26)
γ			0.4783 (0.94)			6.677 (0.99)
c	−0.28	0.008	−0.6935 (−2.54)	−0.05	−0.018	−0.0988 (−6.35)
	France			Japan		
α_0	0.0004 (1.95)	0.0003 (1.44)	0.0017 (1.39)	−0.0001 (−0.38)	−0.0001 (−0.56)	−0.0005 (−1.45)
α_1	0.0289 (0.25)	−0.0281 (−0.58)	0.0664 (4.68)	0.0157 (0.37)	0.0918 (4.21)	0.0890 (3.58)
β_0			−0.0014 (−1.10)			0.0010 (1.33)
β_1	0.1755 (4.96)	0.1247 (3.64)	−0.0617 (−4.34)	0.1143 (4.26)	−0.1083 (−0.88)	−0.0388 (−0.75)
γ			52.3017 (2.02)			2.060 (0.56)
c	−0.15	0.24	−0.2127 (26.15)	0.27	−0.53	−0.1082 (−0.64)

Numbers in parentheses are White robust t-statistics. For model specification see Section 4; models are given by: TAR: $r_t = \alpha_0 + \alpha_1 r_{t-1} I_t + \beta_1 r_{t-1}(1 - I_t) + \varepsilon_t$; where I_t equals one if volume (−1) is above its threshold and zero otherwise. MTAR: $r_t = \alpha_0 + \alpha_1 r_{t-1} I_t + \beta_1 r_{t-1}(1 - I_t) + \varepsilon_t$; where I_t equals one if the change in volume (−1) is above its threshold and zero otherwise. ESTR: $r_t = \alpha_0 + \alpha_1 r_{t-1} + (\beta_0 + \beta_1 r_{t-1})(1 + \exp(-\gamma x_{t-1}^2 - c)) + \varepsilon_t$.

equivalent LSTR model. For Japan, all models generate positive trading returns; this is due to the pronounced (negative) trending behaviour and hence predictable autocorrelation in the series over the forecast horizon. Overall, across all series and models only the LSTR and ESTR models using forecast volume produce positive trading returns, and for three of the four series the LSTR model produces the highest returns. As such we conclude that the LSTR model with forecast volume produces positive returns across all series and is applicable to all series, although on an individual basis greater returns might be found in an alternate model.

5. Summary and conclusion

There has recently been an increased interest in using non-linear models for estimating and forecasting the dynamic behaviour of equity returns. This area of the literature has typically found that such models are able to outperform linear models in terms of their in-sample performance; however, such superiority has rarely extended to the forecasting performance. Several reasons have been advanced for this, including the use of linear forecasting performance metrics, the fact that the out-of-sample data period does not exhibit the same non-linear behaviour as the in-sample period, and that the non-linear model considered is not robust over the whole sample period. The present paper seeks to extend this literature, first, by using daily data for which there should be sufficient non-linear behaviour (should it be present) in the out-of-sample period; secondly by considering forecasting metrics based upon a trading rule, rather than simply the RMSE; and finally, by considering a variable, namely volume, in the estimation and forecasting of which there is an established empirical relationship with returns. More specifically, there is substantial evidence of a negative relationship between volume and future returns, and that low volume is consistent with momentum behaviour in returns and high volume with reverting behaviour.

We consider a smooth-transition regression model where lagged volume is used as the switching variable. Specifically, this allows the dynamics of returns to differ between high and low volume states. The forecasting performance of this model is initially compared to that of an LSTR model where lagged returns are used as the switching variable (and hence the dynamics of the process switch between positive and negative returns, referred to as the LSTAR model), and linear AR and random walk models. Subsequently, we also consider the performance of other popular non-linear models, including the TAR, MTAR and ESTR models.

Table 5
Alternate forecasting evaluation

	TAR	MTAR	ESTR	ESTR with forecasted volume
UK				
RMSE	0.013012	0.013072	0.012945	–
	Trading returns (%) – No short-selling			
Daily return	−0.0398	−0.0358	−0.0385	−0.0001
Cumulative	−52	−47	−50	−0.13
	Trading returns (%) – Short-selling allowed			
Daily return	−0.0493	−0.0373	−0.0428	0.0234
Cumulative	−64	−49	−56	30
US				
RMSE	0.012905	0.012584	0.012651	–
	Trading returns (%) – No short-selling			
Daily return	−0.0150	−0.0002	−0.0311	0.0426
Cumulative	−20	−0.26	−41	56
	Trading returns (%) – Short-selling allowed			
Daily return	−0.0228	0.0001	−0.0521	0.0950
Cumulative	−30	0.13	−68	124
France				
RMSE	0.014479	0.014428	0.014712	–
	Trading returns (%) – No short-selling			
Daily return	−0.0567	−0.0001	−0.0389	0.0206
Cumulative	−74	−0.13	−51	27
	Trading returns (%) – Short-selling allowed			
Daily return	−0.0829	0.0168	−0.0383	0.0625
Cumulative	−108	22	−50	82
Japan				
RMSE	0.012723	0.012810	0.012749	–
	Trading returns (%) – No short-selling			
Daily return	0.0364	0.0404	0.0251	0.0002
Cumulative	48	53	33	0.26
	Trading returns (%) – Short-selling allowed			
Daily return	0.1283	0.1289	0.1059	0.0256
Cumulative	167	168	138	33

RMSE is the root mean square error. Trading returns are given by the following trading rules. The TAR and MTAR strategies are to buy if the next period forecast of returns is greater than zero, and do not buy (sell) if the next period forecast of returns is less than zero. For the ESTR volume model when volume (or forecast volume) is below its threshold value, the strategy is to buy in the next period if returns are forecast positive in the current period, and do not hold (sell) if forecast returns are negative. On the other hand, if volume (or forecast volume) is above the threshold value, the strategy is to not hold (sell) in the next period if the current forecast value is positive and to buy if the forecast value is negative.

The results presented here for four international stock indices suggest that when volume is low returns do indeed exhibit positive serial correlation or momentum behaviour, whilst when volume is high returns appear to exhibit random behaviour or weak reversion. These empirical results are thus consistent with those previously reported in the literature and with the theoretical hypotheses advanced in the recent behavioural finance literature that low volume is associated with momentum behaviour. Turning to the main analytical interest in the paper, the forecasting results based upon a standard forecast metric, namely the RMSE, the out-of-sample performance of the non-linear models is at best comparable, if not marginally inferior, to the linear random walk model. However, when examining forecasting performance in terms of a trading rule based upon the models, the two smooth-transition non-linear models that incorporate volume as the switching variable are superior to the linear random walk and AR models, the alternative univariate (LSTAR) non-linear model and the TAR and MTAR non-linear models, in three of the four cases. Moreover, only the two smooth-transition models that incorporate forecasted volume produce positive trading returns across all series, while the LSTR version produces higher returns than the ESTR model in three of the four series. In summary, the LSTR-forecast volume model appears to provide positive trading returns across all series and could possibly act as a generic trading rule, although on a series-by-series basis an alternate model could generate a higher return, however that alternate model would differ across series. Moreover, this result is robust to small or medium transaction costs, although large transaction costs do appear to negate any trading profits.

Acknowledgements

The author gratefully acknowledges numerous helpful comments on an earlier version of the paper from the editor (Michael Clements), an associate editor and an anonymous referee. Any remaining errors are my own.

References

Baker, M., & Stein, J. (2002). *Market liquidity as a sentiment indicator.* NBER Working Paper.

Balke, N. S., & Fomby, T. (1997). Threshold cointegration. *International Economic Review*, *38*, 627−643.

Balvers, R. J., Cosimano, R. F., & McDonald, B. (1990). Predicting stock returns in an efficient market. *Journal of Finance*, *45*, 1109−1128.

Barberis, N., Shleifer, A., & Vishny, R. (1998). A model of investor sentiment. *Journal of Financial Economics*, *49*, 307−343.

Blume, L., Easley, D., & O'Hara, M. (1994). Market statistics and technical analysis: The role of volume. *Journal of Finance*, *49*, 153−181.

Brennan, M., Chordia, T., & Subrahmanyam, A. (1998). Alternative factor specifications, security characteristics, and the cross-section of expected stock returns. *Journal of Financial Economics*, *49*, 345−373.

Brooks, C. (1997). Linear and nonlinear (non-)forecastability of high-frequency exchange rates. *Journal of Forecasting*, *16*, 125−145.

Campbell, J., Grossman, S., & Wang, J. (1993). Trading volume and serial correlation in stock returns. *Quarterly Journal of Economics*, *108*, 905−939.

Campbell, J. Y., & Hamao, Y. (1992). Predictable stock returns in the United States and Japan: A study of long-term capital integration. *Journal of Finance*, *47*, 43−67.

Campbell, J., Lo, A., & MacKinlay, C. (1997). *The econometrics of financial markets.* Princeton: Princeton University Press.

Chan, K. S. (1993). Consistency and limiting distribution of the least squares estimator of a threshold autoregressive model. *Annals of Statistics*, *21*, 520−533.

Clements, M. P., & Smith, J. (1999). A Monte Carlo study of the forecasting performance of empirical SETAR models. *Journal of Applied Econometrics*, *14*, 123−142.

Coakley, J., & Fuertes, A. M. (2001). Nonlinearities in excess foreign exchange returns. *The Manchester School*, *69*, 623−642.

Cochrane, J. H. (1991). Production-based asset pricing and the link between stock returns and economic fluctuations. *Journal of Finance*, *46*, 209−238.

Conrad, J., Hameed, A., & Niden, C. (1994). Volume and autocovariances in short-horizon individual security returns. *Journal of Finance*, *49*, 1305−1329.

Datar, V., Naik, N., & Radcliffe, R. (1998). Liquidity and asset returns: An alternative test. *Journal of Financial Markets*, *1*, 203−220.

De Gooijer, J. G., & Kumar, K. (1992). Some recent developments in non-linear time series modelling, testing and forecasting. *International Journal of Forecasting*, *8*, 135−156.

Enders, W., & Granger, C. W. J. (1998). Unit-roots tests and asymmetric adjustment with an example using the term structure of interest rates. *Journal of Business and Economic Statistics*, *16*, 304−311.

Enders, W., & Siklos, P. L. (2001). Cointegration and threshold adjustment. *Journal of Business and Economic Statistics*, *19*, 166−176.

Fang, Y., & Xu, D. (2003). The predictability of asset returns: An approach combining technical analysis and time series forecasts. *International Journal of Forecasting*, *19*, 369−385.

Ferson, W. E., & Harvey, C. R. (1993). The risk and predictability of international equity returns. *Review of Financial Studies*, *6*, 527−566.

Granger, C. W. J., & Teräsvirta, T. (1993). *Modelling nonlinear economic relationships.* Oxford: Oxford University Press.

Hong, H., & Stein, J. C. (1999). A unified theory of underreaction, momentum trading, and overreaction in asset markets. *Journal of Finance*, *54*, 2143−2184.

Karpoff, J. M. (1987). The relationship between price changes and trading volume: A survey. *Journal of Financial and Quantitative Analysis*, *22*, 109−126.

Krägler, H., & Krugler, P. (1993). Nonlinearities in foreign exchange markets: A different perspective. *Journal of International Money and Finance*, *12*, 195−208.

Leung, M. T., Daouk, H., & Chen, A. -S. (2000). Forecasting stock indices: A comparison of classification and level estimation models. *International Journal of Forecasting*, *16*, 173−190.

Maasoumi, E., & Racine, J. (2002). Entropy and predictability of stock market returns. *Journal of Econometrics*, *107*, 291−312.

Martens, M., Kofman, P., & Vorst, T. C. F. (1998). A threshold error correction model for intraday futures and index returns. *Journal of Applied Econometrics*, *13*, 245−263.

McMillan, D. G. (2001). Non-linear predictability of stock market returns: Evidence from non-parametric and threshold models. *International Review of Economics and Finance*, *10*, 353−368.

McMillan, D. G. (2003). Non-linear predictability of UK stock market returns. *Oxford Bulletin of Economics and Statistics*, *65*, 557−573.

Newey, W., & West, K. (1994). Automatic lag selection in covariance matrix estimation. *Review of Economic Studies*, *61*, 631−653.

Obstfeld, M., & Taylor, M. (1997). Nonlinear aspects of goods-market arbitrage and adjustment: Heckscher's commodity points revisited. *Journal of the Japanese and International Economies*, *11*, 441−479.

Perez-Quiros, G., & Timmerman, A. (2000). Firm size and cyclical variations in stock returns. *Journal of Finance*, *55*, 1229−1262.

Pesaran, M. H., & Timmermann, A. (1995). Predictability of stock returns: Robustness and economic significance. *Journal of Finance*, *50*, 1201−1228.

Pesaran, M. H., & Timmermann, A. (2000). A recursive modelling approach to predicting UK stock returns. *Economic Journal*, *110*, 159−191.

Shively, P. A. (2003). The nonlinear dynamics of stock prices. *The Quarterly Review of Economics and Finance*, *43*, 505−517.

Teräsvirta, T. (1994). Specification, estimation and evaluation of smooth transition autoregressive models. *Journal of the American Statistical Association*, *89*, 208−218.

Teräsvirta, T., & Anderson, H. M. (1992). Characterising nonlinearities in business cycles using smooth transition autoregressive models. *Journal of Applied Econometrics*, *7*, S119−S136.

Tiao, G. C., & Tsay, R. S. (1994). Some advances in non-linear and adaptive modelling in time-series. *Journal of Forecasting*, *13*, 109−140.

Tong, H. (1983). *Threshold model in nonlinear time series analysis.* New York: Springer-Verlag.

Tong, H. (1995). A personal overview of non-linear time series analysis from a chaos perspective. *Scandinavian Journal of Statistics*, *22*, 399−445.

Wang, C., & Chin, S. (2004). Profitability of return and volume-based investment strategies in China's stock market. *Pacific-Basin Finance Journal*, *12*, 541−564.